RISK MANAGEMENT IN SUSTAINABLE FLEET REPLACEMENT USING CONDITIONAL VALUE AT RISK

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To My Parents

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Bref résumé de la thèse

L'objet de cette thèse est d'analyser comment traiter le problème de renouvellement du parc en tenant compte de la durabilité, tout en se plaçant dans une perspective de gestion du risque.

Cette thèse apporte une double contribution, au niveau de la politique de gestion du parc et à celui de la méthode utilisée pour appliquer cette politique. Au niveau de la politique, elle étudie l'effet de l'adoption de nouveaux véhicules, disposant d'une technologie de pointe, sur le risque et le coût escompté du système de gestion du parc.

Au niveau méthodologique, cette thèse apporte trois contributions. Tout d'abord, elle comporte une étude de la nouvelle formulation du problème du parc en utilisant une programmation stochastique à deux étapes et à multiples étapes et une valeur à risque conditionnelle (CVaR), prenant ainsi en considération l'incertitude dans le processus de décision. En outre, elle élabore une formulation récursive de la CVaR, qui tient compte de la cohérence dans le temps, et elle examine ses propriétés de convergence, dans un cadre dynamique. Enfin, la thèse modélise l'impact sur le profit et le risque de l'utilisation des contrats à option sur le problème de remplacement du parc.

Mots clés : gestion du risque; remplacement ; activités durables ; valeur à risque conditionnelle(CVaR); programmation stochastique

ABSTRACT

The purpose of this thesis is to conduct an analysis of how the fleet replacement problem can be addressed from both sustainability and risk management perspectives, simultaneously.

The contribution of this thesis has two components, in fleet management policy and in the method used to apply it. At a policy level, this thesis addresses the effect of adoption of new technological advanced vehicles on the risk and expected cost of the fleet management system.

At a methodological level, this thesis presents three contributions: First, it studies the new formulation of the fleet problem by using a two stage and a multi stage stochastic programming and conditional value at risk (CVaR), which accounts for the uncertainty in the decision process. Second, it models a recursive formulation of CVaR, which takes into account the time consistency, and studies its convergence properties, in a dynamic setting. Third, it models the impact on profit and risk from using option contracts on the fleet replacement problem.

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Résumé détaillé de la thèse

Alors que nous entamons le 21e siècle, le monde est confronté à deux défis qui vont définir notre avenir : la perspective de la crise d'un changement climatique catastrophique et la lutte contre la pauvreté dans le monde. Nous sommes confrontés en outre, à court terme, à la plus sévère crise financière et économique depuis les années 80. La crise financière a été causée par une gestion inappropriée du risque dans le secteur financier. De même, la gravité de la crise climatique va dépendre de notre gestion des risques liés aux gaz à effet de serre. Ces risques toutefois sont fondamentalement différents. Nos actions pour juguler la crise financière vont se traduire par la perte minime ou un peu plus conséquente de quelques points de PIB et par une durée de crise d'un an ou deux ou d'une décennie. Les conséquences des erreurs dans la gestion de la crise climatique sont d'une toute autre ampleur, et sont susceptibles d'avoir des suites majeures et irréversibles pour la vie sur cette planète.

Chaque fois que nous brûlons des combustibles fossiles tels que le gaz, le charbon ou le pétrole, du CO_2 se répand dans l'atmosphère. Dans le cycle naturel du carbone, le CO_2 est réabsorbé par des plantes et des arbres. Mais nous brûlons du pétrole contenant du CO_2 coincé sous la surface de la terre depuis des millions d'années, et nous agissons si vite que les plantes et les arbres vivant actuellement n'ont aucune chance de l'absorber (et le déboisement des forêts tropicales n'améliore pas les choses). Ce supplément de CO_2 dans l'atmosphère a pour effet d'augmenter la température globale de la planète (le réchauffement planétaire). Alors que les températures moyennes mondiales augmentent, au quotidien le climat est en train de changer de manière imprévisible (depuis les inondations et les ouragans jusqu'aux vagues de chaleur et aux sécheresses). Afin de tenter de réduire le risque de phénomènes climatiques toujours plus extrêmes, il nous faut réduire notre combustion de carburant fossile. Aussi, durant les 25 dernières années, les gouvernements ont-ils commencé à admettre que le développement économique actuel ne pouvait pas se poursuivre sans avoir une incidence significative sur les générations futures. Par exemple, aujourd'hui, nous voyons l'émergence d'une tendance entre les pays à l'échange de quotas d'émission pour la gestion des gaz à effet de serre (par exemple, Sainathan et al., 2013).

Le Rapport Brundtland (WCED, 1987) a reconnu que le développement économique actuel ne pouvait plus compromettre les besoins de développement des générations futures. L'objectif de ce concept de développement durable a été d'encourager les gens à s'impliquer pour comprendre la façon dont le développement économique peut affecter à la fois l'environnement et la société. Ainsi, la question de savoir comment répondre aux besoins du présent sans compromettre la capacité des générations futures de répondre à leurs besoins comporte d'importants aspects environnementaux, économiques et sociaux. Il s'agit de la question de la durabilité, qui constitue sans doute le plus grand défi de notre génération et de la suivante (par exemple, Schiffer, 2008). Dans l'environnement mondial d'aujourd'hui, afin de relever ce défi, il faut un engagement du secteur privé et du secteur public, des organisations non gouvernementales et enfin de tous les individus. En raison de l'émergence de ces préoccupations, il y a une pression sur les entreprises pour qu'elles diminuent leur impact sur l'environnement, qu'elles s'appliquent à extérioriser un triple résultat (au niveau des gens, du profit et de la planète) et par conséquent qu'elles réduisent leurs émissions de dioxyde de carbone. Les activités de base qui contribuent à ces émissions sont la fabrication et le transport de produits, le recyclage, la refabrication des produits utilisés, et la conception de nouveaux produits (Kleindorfer et al., 2005).

En outre, la volatilité récente du prix des carburants fossiles et l'accroissement des préoccupations liées au réchauffement planétaire ont mis en évidence la nécessité de réduire la consommation d'énergie et de carburants fossiles. Le secteur du transport constitue une source importante d'augmentation des émissions de CO2 (par exemple, Schiffer, 2008). La raison du haut niveau de ces émissions réside dans une forte dépendance à l'égard des carburants fossiles. Par conséquent, les nouvelles technologies, telles que les véhicules hybrides et les véhicules électriques (VE) sont considérées comme une alternative permettant de réduire les niveaux de consommation de carburants fossiles et d'émission de gaz à effet de serre. Même lorsqu'on tient compte de la génération d'énergie utilisée dans la chaîne d'approvisionnement pour la production de l'électricité utilisée afin de charger les batteries des VE, les émissions totales des VE demeurent inférieures à celles issues des véhicules utilisant des carburants fossiles, surtout dans les pays développés.

Les VE possèdent deux avantages par rapport aux véhicules à moteur à combustion interne. Le premier et le plus important, c'est leurs plus faibles émissions de CO2, car l'électricité fournie pour charger les batteries des VE peut être générée à l'aide de sources d'énergie renouvelables, telles que l'énergie éolienne ou solaire. Les VE disposent d'un deuxième avantage : ils sont moins tributaires des incertitudes liées à l'approvisionnement en carburants fossiles (telles que les fluctuations du prix du pétrole brut) car le prix de l'électricité par kilomètre est nettement inférieur à celui de l'essence ou du diesel.

Malgré les points forts des VE cités ci-dessus, nous ne sommes qu'au début du processus d'adoption de ces véhicules, pour deux raisons principales. La première concerne «l'anxiété liée à l'autonomie», c'est-à-dire la peur associée au kilométrage limité effectué par les VE avant d'être obligé de les recharger (par exemple, Eberle et Helmolt, 2010). Actuellement, les VE chargés disposent d'une autonomie limitée à environ 150 km, et il faut compter plusieurs heures pour recharger les batteries. Le coût de la batterie constitue la deuxième raison. La batterie est la composante la plus onéreuse d'un VE, d'où la différence du coût d'un VE par rapport à celui d'un véhicule utilisant un carburant fossile. En outre, même si les coûts de fonctionnement d'un VE sont bien inférieurs à ceux d'un véhicule utilisant un carburant fossile, les coûts fixes (liés à la location ou à l'acquisition) sont actuellement très élevés.

Pour contribuer à l'amélioration du transport durable, la gestion du parc joue un rôle important de deux manières. D'abord, elle exerce un effet économique direct sur l'investissement, la maintenance et les frais de fonctionnement. En second lieu, elle aide à diminuer les émissions en dioxyde de carbone de l'entreprise. En outre, même si la comparaison des coûts relatifs des différents types de véhicule est de bien des points de vue une question d'optimisation assez évidente, il existe d'autres difficultés à résoudre qui font de cette gestion un sujet de recherche intéressant pour la gestion durable (voir la Figure 1.1). Parmi ces difficultés, figurent les incertitudes quant au prix du marché de différentes sources d'énergie, au prix des émissions de carbone, à la consommation de carburant, et au kilométrage effectué par les véhicules. Donc, il faut impérativement examiner le problème de la gestion durable d'un parc sous l'angle des incertitudes à l'aide de méthodes de gestion du risque (Figure 1.1).

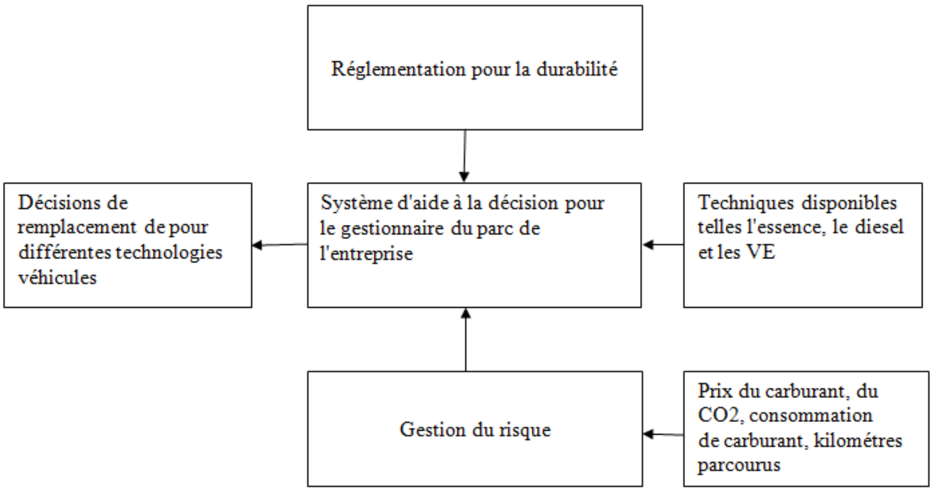

Figure 1.1 Système d'aide à la décision pour le gestionnaire de parc

La thèse est ensuite résumée, les conclusions principales sont tirées, et l'accent est mis sur les contributions principales de cette thèse. En premier lieu, nous examinons les contributions principales. La contribution de cette thèse peut être divisée en deux composantes : la politique de gestion de parc et la méthode utilisée afin d'appliquer cette politique.

Contributions

Au niveau de la politique, cette thèse examine l'effet de l'adoption de nouveaux véhicules disposant d'une technologie de pointe sur le risque et le coût escompté du système de parc des entreprises. L'idée de traiter cette question provient du besoin d'étudier, d'un point de vue économique, l'adoption de nouveaux véhicules de pointe dans un grand nombre d'entreprises en Europe et aux Etats-Unis. Etant donné que les VE n'en sont encore qu'à leurs débuts en termes de développement et du fait qu'ils nécessitent des investissements d'un coût élevé, la présente étude traite cette question du point de vue du risque, une approche qui n'a jamais été utilisée auparavant.

Au niveau méthodologique, cette thèse apporte trois contributions. Tout d'abord, aux chapitres 3 et 4, elle présente une nouvelle méthode d'approche du problème du parc en s'appuyant sur la programmation stochastique à deux étapes et à étapes multiples et sur la CVaR, qui tient compte des incertitudes dans le processus de prise de décision. En d'autres termes, l'une des contributions des chapitres 3 et 4 est l'examen du risque et la réduction maximale du coût à l'aide de la CVaR, dans un modèle de programmation stochastique, dans le cadre de la fonction objective de l'entreprise, ce qui n'a jamais encore été étudié dans la littérature. Comme, précisément, l'objectif de ce programme stochastique est de réduire au maximum le coût et le risque, nous avons simultanément réduit au maximum la moyenne pondérée des coûts totaux escomptés et la CVaR. Ce qui veut dire qu'en changeant un paramètre de la relation exogène pour différentes combinaisons des coûts totaux escomptés et de la CVaR, les risques à l'horizon du plan sont réduits au maximum, selon qu'on mette davantage l'accent sur le coût ou sur le risque.

En second lieu, au chapitre 4, une formulation récursive de la CVaR est modélisée. Elle tient compte de l'uniformité dans le temps et des propriétés convergentes dans un cadre dynamique. En effet, notre approche diffère de celle de Shapiro (2009, 2011), dans laquelle la configuration conditionnelle du risque a été utilisée afin de satisfaire le principe de l'uniformité dans le temps, alors que nous fournissons une formulation récursive de la CVaR pour une arborescence des scénarios, calculant explicitement la CVaR du nœud parent comme une fonction de la CVaR et les attentes conditionnelles escomptées du coût extrême des nœuds enfants respectifs. Notre approche diffère également de l'approche objectif-centile de Boda et Filar (2006), qu'ils appliquent afin d'examiner l'uniformité dans le temps. Donc, notre contribution méthodologique exposée au chapitre 4 revient à suggérer une nouvelle formulation de la CVaR uniforme dans le temps.

Puis, au chapitre 5, le modèle est étendu dans un cadre dynamique pour la CVaR afin de prendre en compte la souplesse dans le problème de remplacement du parc en ayant recours à des contrats avec différentes options. En effet, il existe une lacune dans la littérature, qui consiste à considérer le problème du remplacement durable du parc en tenant compte de la flexibilité inhérente aux contrats de location du fait de l'existence d'incertitudes en ayant recours à l'analyse par options réelles. Notre approche dans cette thèse diffère de celle de Kleindorfer et al. (2012), en termes de paramètres stochastiques dans le modèle et d'options différentes pour les contrats de location. Ceci constitue également une nouvelle approche dans la littérature, qui tient compte de l'interaction entre l'utilisation de contrats et la CVaR dans le système de gestion du parc, ce qui n'avait pas été analysé jusqu'à présent. En outre, au chapitre 5, pour l'évaluation des contrats avec option, on examine l'impact des changements technologiques concernant les batteries des VE. Puis, les conclusions principales sont tirées.

Conclusion

Au chapitre 2, nous avons effectué une analyse approfondie de la littérature pour plusieurs approches du problème de remplacement de l'actif que constitue le parc. Plus précisément, lorsque nous examinons les décisions des gestionnaires de parc dans les entreprises portant sur le remplacement des véhicules conventionnels, et l'impact des technologies nouvelles sur l'adoption de politiques de remplacement optimal, le gestionnaire du parc doit aborder plusieurs questions essentielles. Tout d'abord, quelles sont les technologies s'appliquant aux véhicules automobiles qui réalisent les meilleures performances en termes de coût-efficacité ? Ensuite, quel est l'impact des aléas du marché sur les décisions de remplacement des véhicules ? Enfin, quelles sont les meilleures pratiques pour le remplacement des véhicules dans l'avenir ? Le modèle que nous proposons au chapitre 2 peut pallier certains inconvénients des méthodes actuelles de remplacement. En premier lieu, il tient compte de la variabilité du coût de fonctionnement (coût d'exploitation) des véhicules. En effet, la majorité des paramètres du modèle sont liés au temps et les paramètres du coût se divisent en fixes et variables. En conséquence, l'utilisation annuelle escomptée (le kilométrage total annuel) est considérée comme une variable chaque année. De plus, le coût des émissions de CO2 (par exemple, Moreira et al., 2010) est également pris en compte. En outre, à la différence de la plupart des articles publiés dans la littérature, l'option de location est considérée comme un moyen de financement des véhicules dans le système de gestion du parc, ce qui est couramment utilisé dans la plupart des systèmes de logistique commerciale. En prenant en considération la location des nouveaux véhicules au début de chaque année pour une durée donnée (4 à 5 ans), un bon nombre de problèmes vont être résolus en raison de la jeunesse de la structure du système de parc : ceux concernant l'âge optimal (la vie économique) des véhicules et le rapport entre l'âge et l'utilisation. Toutefois, le modèle nécessite un certain nombre de données déjà réalisées et prévisionnelles, telles que le prix du carburant, la consommation de carburant et le prix du CO2, l'utilisation des véhicules sur plusieurs années. Ces données doivent être rassemblées, actualisées et traitées en utilisant une base de données moderne. Cette base de données, allant de pair avec le modèle proposé, donne un système d'aide à la décision pour une gestion stratégique du parc de toute entreprise de transport.

Aux chapitres 3, 4, et 5, les facteurs de risque examinés sont le prix du carburant et du CO2, le kilométrage effectué, et la consommation de carburant. Au chapitre 3, nous avons envisagé une répartition différente des facteurs de risque par rapport à celle retenue aux chapitres 4 et 5, car les modèles sont étudiés dans des contextes différents. En outre, au chapitre 3, nous avons examiné la location d'un seul véhicule de marques différentes, et au chapitre 4, nous avons analysé l'interaction entre les véhicules d'un parc avec différentes capacités. Enfin, tous les véhicules utilisés dans le parc sont loués.

Au chapitre 3, nous avons évalué l'importance des facteurs de risque pour les VE et les véhicules diesel et comparé la valeur de la CVaR des VE avec la technologie diesel pour chaque cas de facteur de risque. Les résultats indiquent que, lorsqu'on examine chaque processus stochastique séparément, le facteur de risque le plus important pour un véhicule diesel est le kilométrage parcouru, le deuxième facteur étant la consommation de carburant, et le troisième le prix du carburant. Dans le cas des VE, le kilométrage parcouru constitue le facteur de risque le plus important, suivi du prix du carburant et enfin du prix du CO2. En outre, pour chaque processus stochastique portant sur le prix du carburant, le kilométrage parcouru et la consommation de carburant, la valeur CVaR pour les VE est inférieure à celle des véhicules à carburant fossile, à certaines conditions. Par ailleurs, lorsqu'on examine tous les processus stochastiques conjointement, la location d'un véhicule diesel plutôt que d'un véhicule électrique diminue le coût total escompté et augmente le risque associé en raison de l'incertitude sur le prix du CO2, le prix du carburant, le kilométrage parcouru, et la consommation de carburant. De plus, l'examen conjoint de tous les processus stochastiques montre que le risque du modèle dans son ensemble est inférieur à la somme des risques de chaque processus stochastique. Enfin, en comparant le coût total au kilomètre pour chaque scénario de distance parcourue (par mois) et en introduisant d'autres facteurs d'incertitude dans le modèle d'aide à la décision, on peut arriver à la conclusion que, pour les véhicules à kilométrage élevé, le VE constitue le choix le plus approprié.

Au chapitre 4, nous avons examiné la question de la gestion du risque lorsqu'on a recours, dans le système de gestion du parc, à différents types (capacité) de véhicules, utilisant différentes technologies. La capacité constitue une caractéristique importante. En effet, selon l'usage, les techniciens de l'entreprise doivent transporter du matériel et, par conséquent, disposer d'un véhicule d'une capacité suffisante. Nous examinons trois types de véhicule de capacités différentes. Ainsi, les ingénieurs électriciens ont besoin de fourgonnettes dont la capacité de charge est suffisante. Nous considérons que les petites fourgonnettes pèsent 300 kg, alors que les fourgonnettes pèsent 500 kg et ont une plus grande capacité. Les fourgonnettes de taille moyenne peuvent être utilisées pour tous les usages, mais il y en a très peu, car elles sont nettement plus onéreuses à l'achat et au niveau de l'entretien. Nous avons effectué une analyse typologique pour chaque type de véhicule. Dans ce cas, chaque véhicule génère un risque différent, selon ses caractéristiques. Pour cette raison, nous les avons classés (regroupés) à partir de deux facteurs de risque importants, le kilométrage parcouru et la consommation de carburant, puis nous avons étudié le comportement de chaque catégorie, pour chaque type de véhicule et chaque type de technologie employée. En outre, nous avons comparé le comportement de chaque groupe au cas où il n'y a pas de regroupement (regroupement combiné). Au chapitre 4, nous avons également examiné les véhicules hybrides. Les principaux résultats du modèle sont les suivants. a) Pour les regroupements à faible kilométrage (750 km/mois) et à kilométrage moyen (1500 km/mois), en ce qui concerne le rendement en carburant pour tous les types de véhicule, le gazole est le choix prépondérant pour réduire au maximum le risque ou le coût. b) L'essence est le choix le plus approprié pour réduire au maximum le risque et le coût dans les regroupements à faible kilométrage et à kilométrage moyen de différents types de véhicules. c) Les modèles hybride/essence et hybride/diesel, qui ont également été pris en compte dans le modèle, ne peuvent pas concurrencer la technologie du diesel en termes de coût-efficacité en raison du coût élevé de la location ou de l'achat. Leur taux de pénétration selon les différents types de véhicules est de 1 à 3 %. d) Les VE font généralement partie des regroupements à kilométrage élevé de véhicules de différentes capacités. e) Pour toutes les capacités, le risque par véhicule est réduit par le regroupement, par rapport aux regroupements combinés. Toutefois, on ne peut pas diminuer le coût prévu par véhicule en effectuant des regroupements des catégories à kilométrage élevé. La raison est la suivante : l'adoption des VE, pour les regroupements à kilométrage élevé, augmente le coût prévu par véhicule. En outre, nous avons un certain pourcentage de véhicules diesel et à essence, et ces véhicules ne sont pas économiques pour des kilométrages élevés.

Au chapitre 5, nous avons traité une autre question importante, relative à la location des véhicules. Les établissements de location et les (grandes) entreprises négocient des contrats de location et les conditions de ces derniers. Ensuite, après avoir négocié avec plusieurs établissements de location, une entreprise choisit celui qui obtient le contrat. Actuellement, un contrat de location commercial a généralement une durée de quatre ans dans le cas de l'entreprise que nous avons analysée. Toutefois, on s'attend à beaucoup de développements avec les VE et les modèles améliorés qui vont devenir disponibles dans les années à venir (par exemple, l'amélioration des batteries). Aussi, les entreprises peuvent-elles préférer attendre afin d'obtenir un meilleur modèle de VE d'ici quelques années. Par ailleurs, d'autres technologies comme le diesel et l'essence, en raison des interventions de l'Etat, de l'augmentation du prix des carburants fossiles, entre autres causes, seront peut-être moins économiques à l'avenir. On pourrait également négocier des contrats de location d'une durée plus courte pour les VE. Toutefois, cette approche donne lieu à une dépréciation plus importante et entraîne donc des frais mensuels de location plus élevés pour les VE, ce qui les rend moins économiques. En revanche, pour les voitures diesel et à essence, si une forte augmentation du prix des carburants fossiles intervenait, le coût de fonctionnement deviendrait élevé, et des contrats d'une durée de deux ans pourraient être judicieux. Un autre aspect qui mérite d'être souligné : dans le cas des véhicules diesel et à essence, en raison de leur technologie mature, le pouvoir de négociation du prix de location par les entreprises est plus important, même pour les contrats d'une durée relativement courte. Pour cette raison, nous avons étendu le modèle en incluant plusieurs contrats à option. La première formule, c'est le contrat de base sans option d'une durée de quatre ans. Toutefois, si la voiture est rendue, les pénalités sont très élevées. La deuxième possibilité, c'est la location du véhicule avec l'option de le rendre en versant de faibles pénalités. Enfin, la troisième possibilité, c'est de louer le véhicule avec une option d'échange permettant de rendre le véhicule et d'en choisir un autre en s'acquittant de faibles pénalités. Afin d'évaluer les contrats de location mentionnés ci-dessus, nous pouvons recourir à la théorie des options réelles avant de développer notre modèle dynamique, fondé sur nos recherches précédentes, pour déterminer le nombre optimal de voitures pendant la période de location. Nous avons examiné également le développement technologique des batteries des VE prévu au cours de la durée des contrats dans une optique de politique de remplacement optimale. Les principales conclusions de cet examen sont les suivantes. a) Le recours à des contrats assortis d'options diminue la CVaR globale et le coût escompté. En d'autres termes, le recours à tous les contrats dans le système de gestion du parc permet de minimiser le coût total et la CVaR. b) La technologie prédominante pour la location est celle des VE lorsqu'on prend en compte l'effet de la technologie. Cependant, lorsqu'on prend en considération le modèle sans le développement technologique des VE, la technologie choisie de façon prédominante est celle du diesel. On retient la technologie de l'essence lorsqu'on estime qu'il n'y aura pas de développement technologique pour les VE dans un petit nombre de cas. c) Si nous comparons également la CVaR et le coût escompté dans deux cas avec et sans changement technologique au niveau des VE, pour chaque coefficient correspondant pour le rapport entre la valeur et le prix d'une option, nous arrivons à la conclusion qu'il y a une diminution des valeurs de la CVaR et du coût escompté, pour toutes les valeurs du coefficient, lorsque le développement technologique des VE est pris en compte. En outre, à partir des précédentes recherches, nous sommes arrivés à la conclusion que plus il y avait de

VE dans le système de gestion du parc plus la CVaR était faible. Ainsi, la réduction de la CVaR est plus forte que le coût escompté.

CHAPTER 1

INTRODUCTION

During the last 25 years governments have started to recognize that the current economic development could not be sustained without significant impacts upon future generations. For example, nowadays we see the emerging trend among nations towards the use of emissions trading in managing greenhouse gases (e.g., Sainathan et al., 2013).

The Brundtland Report (WCED, 1987) recognized that economic development which is taking place today could no longer compromise the development needs of future generations. This concept of sustainable development aimed to encourage people to be involved for realizing on how the economic development can affect both on the environment and on the society. So, the question of how to meet the needs of the present without compromising the ability of future generations to meet their needs which has important environmental, economic, and social dimensions, i.e., sustainability, is arguably the greatest challenge of our generation and the next (e.g., Schiffer, 2008). Due to these emerging concerns, companies are under pressure to reduce their impact on the environment, to engage in measuring the triple bottom line (people, profit, and planet), and consequently, to reduce their resulting carbon footprint. Basic activities that contribute to this footprint are the production and transport of products, recycling, remanufacturing of used products, and designing of new products (Kleindorfer et al., 2005).

In addition, the recent volatility of fossil fuel prices, and the increasing concerns regarding global warming, have highlighted the need to reduce fossil fuel energy consumption. The transportation sector is an important cause of increasing CO_2 emissions (e.g., Schiffer, 2008). The reason for these high emissions is the dependency on fossil fuels. As a result, new technologies such as hybrid vehicles and electric vehicles (EVs) are considered an alternative to reduce fossil fuel consumption and greenhouse gas emission levels. Even when considering the power generators used in the supply chain for electricity production to charge EV batteries, the total emissions by EVs are still lower than emissions from fossil fuel vehicles, especially in developed countries.

EVs have two advantages over internal combustion engine vehicles: the first, and most important, is lower CO_2 emissions, as the electricity supplied for the EV battery may be generated by renewable energy sources, such as wind or solar power. The second is that EVs are not affected as much by the uncertainties arising from the supply side of fossil fuels (such as fluctuations in crude oil prices) as the cost of electricity, per mile, is less than that of gasoline or diesel.

Despite the aforementioned benefits of EVs, their adoption is still in the infancy stage, for two main reasons. The first is "range anxiety", which is the fear associated with the limited mileage these vehicles may be driven before they need to be recharged (e.g., Eberle and Helmolt, 2010). Currently, EVs have a limitation of about 100 miles on a single charge, and it takes several hours to charge their batteries. The second factor is the cost of the battery, which is the most expensive component of EVs, resulting in the cost difference between EVs and fossil fuel vehicles. In addition, although the running cost of an EV is much less than that of a fossil fuel vehicle, the fixed cost (both with leasing or ownership) is currently very high.

Fleet management is an important effort to help in improving sustainable transportation in two ways. First, it has a direct economic effect on investment, maintenance, and operating costs. Second, it is able to help in reducing the carbon footprint in the company. In addition, whereas the comparison of the relative cost of the different types of vehicles is, in many aspects, a relatively obvious optimization problem, there exist additional complexities that make it an interesting research topic for sustainable management (see Figure 1.1). These include the uncertainties in market prices for various sources of energy, carbon emission prices, fuel consumption, and the mileage driven by the vehicles. Thus, it is essential to view the problem of sustainable fleet management from an uncertainty perspective using risk management methodologies (Figure 1.1).

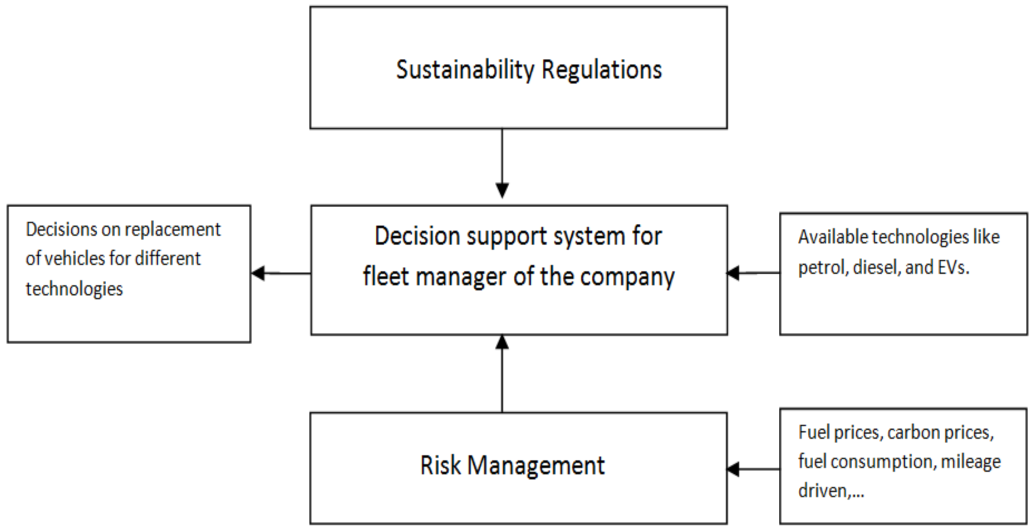

Figure 1.1 Decision Support System for Fleet Manager

The purpose of this thesis is to conduct an analysis of how the fleet replacement problem can be addressed from both sustainability and risk management perspectives, simultaneously.

The contribution of this thesis has two components, in fleet management policy and in the method used to apply it. At a policy level, this thesis addresses the effect of adoption of new technological advanced vehicles on the risk and expected cost of the fleet management system.

At a methodological level, this thesis presents three contributions: First, it studies the new formulation of the fleet problem by using a two stage and a multi stage stochastic programming and conditional value at risk (CVaR), which accounts for the uncertainty in the decision process. Second, it models a recursive formulation of CVaR, which takes into account the time consistency issue, and is called Recursive Expected CVaR (RECVAR). It is a new time consistent risk measure in the sense that it takes account the risks that happen in the future. Third, it models the impact on profit and risk from using option contracts on the fleet replacement problem.

This thesis is organized as follows. In Chapter two we provide a comprehensive literature review on the asset (fleet) replacement problem and a general introduction and survey of risk management and its applications especially in operations as well as a literature survey on sustainable operations. Specifically, we introduce a general classification of fleet replacement models, and we consider different approaches for replacement decisions. Then, we focus on parallel replacement problem, which is the core fleet replacement idea of this thesis. In addition, we extend the parallel replacing model, in the leasing context. In addition, we extend the parallel replacing model, in the leasing context. In the risk management section we introduce CVaR and VaR and their properties and then we provide a broad literature on sustainable operations. Finally, we conclude the Chapter by practical challenges for the replacement problem.

In Chapter three, first we provide a two-stage stochastic programming model for the replacement policy. This is a static policy in which decisions are made at the first stage and we study one type of vehicle with different brands. We also present some analytical results for comparing the CVaR of fossil fuel vehicles and EVs, taking into account the volatility in CO_2 and fuel prices, fuel consumption, and mileage driven. Finally, we validate the analytical results by a real case study and we conclude the Chapter.

In Chapter four we extend the work in Chapter three to a multi stage setting. In this context the decisions are updated at every period in which the interaction between different types of vehicles with different capacities by using clustering analysis has been studied. As a methodological contribution, in this Chapter, we present a new recursive formulation of CVaR which is time consistent in a dynamic setting.

Additionally, by using clustering analysis we consider the portfolio effect of using different technologies, on the fleet system, on CVaR, and on the expected cost. Finally, we present the analytical results for considering the portfolio effect and time consistency of new formulation of CVaR and we apply the model in a real case study.

In Chapter five we extend the work presented in Chapter four by considering flexible leasing contracts. In this framework decisions are updated at every period with different options. Indeed, in this context, we have a full flexibility through using contracts with different options. First, we provide a literature review on using real options. Then, we extend our previous model, in Chapter four, by using CVaR and different options contracts which are base (contract with no option), return, and swap. In Chapter five we also consider the technological development of the batteries expected during the planning horizon on the optimal replacement policies. Finally, we present analytical results to explain how option contracts affect the CVaR, and total expected cost which we apply in the analysis of a case study.

CHAPTER 2

Literature Review

Le chapitre 2 offre une analyse détaillée de la littérature consacrée au problème du remplacement de l'actif que constitue le parc. Plus précisément, nous apportons une classification générale des modèles de remplacement du parc et nous étudions différentes approches de la décision de remplacement. Puis nous abordons le problème du remplacement parallèle, qui est l'idée de base de cette thèse. En outre, nous étendons le modèle du remplacement parallèle, dans le contexte de la location et enfin nous concluons ce chapitre en exposant des difficultés pratiques liées au problème du remplacement.

In this Chapter first we provide a broad literature survey on the sustainable development and its important role in the operations and supply chain management fields. Then we consider a detailed history of asset replacement problem which is the core idea of this thesis. Finally, we study a general introduction and survey of risk management and its applications especially in operations.

Section 2.1 presents the literature survey on sustainable operations. Specifically, we consider the concepts of sustainable development, green and sustainable supply chains, closed supply chains, and reveres logistics which are mostly mentioned in the literature.

In Section 2.2 we provide a classification of different asset replacement models, which are broadly classified into serial and parallel models. In addition, we describe the different approaches to modelling the asset replacement problem. Then, we summarize and discuss the methods used to solve the parallel asset replacement problems and suggest a new formulation to address some of their drawbacks. In addition we summarize our insights from our literature review on the asset replacement problem and we analyze, from a practical perspective, the limitations of the asset replacement models. In section 2.3 we provide a general discussion of risk management issues in operations and supply chains. Moreover, we define the concepts of risk measuring, coherent risk measure, VaR, and CVaR. Finally, we conclude the Chapter in Section 2.4.

2.1 A Literature Survey on Sustainable Operations

The ideas of sustainable development were developed in the 1987 Brundtland Report (WCED, 1987), also known as "Our Common Future". It defined sustainable development as a development that meets the needs of the present without compromising the ability of future generations to meet their own needs. This is a definition that is still widely mentioned in the literature (e.g., Stubbs and Cocklin, 2008). The studies have considered the impact of environmental concerns on operations and the effect on existing operational strategies, such as cost, quality, delivery and flexibility. Increased attention of environmental practices is based around the three P's of people, profit and planet, sometimes referred to as the 'Triple Bottom Line' (3BL), (Kleindorfer et al., 2005) or the 'Three Pillars' (White and Lee, 2009). Finally, sustainable development tries to compromise the conflict between economic, environmental and social issues.

A sustainable Operations Management (OM) is defined as the sum of abilities and concepts which allow companies for implementing and managing its business processes in order to obtain some competitive return in its capital assets, without compromising the needs of the inner and outer interested firms, in addition to taking into account the impact of their operations on people and environment (Kleindorfer et al., 2005). As a result, the future operation models will contain a set of extra measures according to environmental and political criteria, such as agility and sustainability by the firm, in the future and efficient utilization of scarce resources (Bayraktar et al., 2007). The inclusion of the sustainability should be accomplished by taking into account strategies and actions which will meet the needs of the companies and of their several stakeholders, thus protecting, maintaining and improving the human and natural resources which may be necessary in the future (Labuschagne et al., 2005)

Studies which are mentioned in the OM literature are mostly in the areas of operations strategy, supply chain management, performance management, performance measurement and service operations. Base on Labuschagne et al. (2005) among the potential research topics, sustainability is more explicitly included, emphasizing that the studies on OM will be relevant for analyzing world issues and global changes.

Next, we consider the literature related to green and sustainable supply chain management, reverse logistics which are studied more in the literature of sustainable operations.

2.1.1 Green Supply Chain Management

Environmentally business practices have been receiving increasing attention from both researchers and practitioners. The number of organizations considering the integration of environmental practices into their strategic plans and operations is continuously increasing (Sarkis, 2003). Several initiatives have provided incentives for organizations to become more environmentally friendly. The concepts related to supply chain environmental management (SCEM) or greening the supply chain are usually taken into account by industry as screening suppliers for environmental performance and then doing business with only those that meet the predefined standards (Rao, 2002).

In green management practices concepts such as environmental management systems, cleaner production, and eco-efficiency have been adopted. The factors creating the competitive advantage through environmental performance have been recognized as market expectations, risk management, regulatory compliance and business efficiency (Zhu and Sarkis, 2005). Green supply chain management (GSCM) has a fundamental role in ensuring that all of the aforementioned factors are addressed (Hutchison, 2003). Environmental impact happens at all stages of a product's life cycle. Therefore GSCM has emerged as an important new concept for companies to obtain profit and market share objective by reducing the environmental risks and impacts while increasing their ecological efficiency (Van Hock, 1999).

2.1.2 Sustainable Supply Chain Management (SSCM)

The most accepted definition for Sustainable Supply Chain Management (SSCM) is that the process of managing the SCM activities with consideration for environmental, economical and social issues for improving the long-term economic goals of individual organization and its supply chains (Farahani et al., 2009)

The SSCM activities that have been considered in the literature review are represented in Figure 2.1. A brief explanation has been provided for each activity as it is related to sustainability concept.

Figure 2.1 Sustainable Supply Chain activities

The first activity in implementing SSCM is creating sustainable design strategies for the product and for the package. This activity also includes designing products such that it could be recycled or remanufactured. Sustainable design will lead to achieve a successful recycling process. Indeed, it assists firms to obtain customers' respect, save money and lead to better products (Toupin, 2001). The interest in implementing environmental packaging, choosing appropriate raw materials matching to environmental standard, and attention for recycling were observed in the middle of the 90's, (Webb, 1994).

Production is the second activity that has an important role in creating SSCM. Environmental production can be obtained by using clean production method, new technology, and reducing raw materials and resources to reach low input, high output and low pollution (Baojuan, 2008). Lean manufacturing or the Just-in-time technique is the first production strategy that obtained environmental goals or named as environmental production (Drumwright, 1994).

Marketing is also a very important activity in developing and implementing SSCM. In order to achieve sustainable marketing, organizations should take into account biological balance, and pay more attention to environmental protection (Baojuan, 2008). Rao (2002) argues that management of wastes in sustainable marketing can lead to cost savings and improved competitiveness. In addition, it helps organizations improve their relationship with customers, suppliers, and other partners.

Sustainable transportation is another important element in developing effective SSCM. Many factors including fuel sources, type of transport, infrastructure, and operational and management practices should be considered in developing zero pollutant transportation systems. Kam et al. (2006) mention that these factors and the dynamics that connect them, determine the environmental impact generated in the transportation logistics phase of the supply chain.

Developing SSCM also needs implementing sustainable purchasing strategies. Liang & Chang (2008) mention that sustainable purchasing leads to reducing waste and hazardous materials by using environmental raw materials. Furthermore, sustainable purchasing plays a significant role in SSCM because it helps organizations in reducing the source of pollution and waste by using strategies such as recycling, scrapping, dumping, or sorting and using biodegradable packaging (Min and Galle, 1997).

2.1.3 Closed Loop Supply Chains and Reverse Logistics

Materials, products, components, equipment may go backwards in the supply chain. Sometimes, we have faced with products being reworked during manufacturing due to unsatisfactory quality, or with good materials or components being returned from the production floor because they were leftover after production. Defective products may be detected after they have entered the supply chain leads to a pull back of products through the chain.

So, products may reverse direction in the supply chain for several reasons which are (e.g., De Brito and Dekker, 2004): (1) manufacturing returns (2) commercial returns (3) product recalls (4) warranty returns (5) service returns (6) end-of-use returns (7) end-of-life returns. In a normal situation, a product is developed and goes into production through the supply chain with the purpose of reaching a customer. However, the product may go back in the chain. From this moment on, the chain does not deal any longer with supply alone, but also with recovery-related activities. We call it as the supply chain loop. As a result, there exists a possible integration of forward and reverse flows. In addition, it includes both the closed loop supply chains, in which the reverse flow goes back to the original user or original function, as well as open loop supply chains. Next, we consider the concept of reverse logistics.

The main goal in reverse logistics is the collection of the products in order to be recovered and the redistribution of the processed goods. Even though this problem is like to the standard forward distribution problem, there exist some differences. Other definition of reverse logistics is defined by (Carter and Ellram, 1998): Reverse Logistics is a process in which companies can be more environmentally efficient through recycling, reusing and reducing the amount of materials used. Indeed, it can be viewed as the reverse distribution of materials between channel members. A more complete view of reverse logistics contains the reduction of materials in the forward system such that fewer materials flow back, reuse of materials is feasible and recycling is possible.

Next, we provide a detailed literature review on asset replacement problem which is the core idea of this thesis and then we find the gap which is not addressed in the literature of sustainable operations and asset replacement problem in the summary of this Chapter.

2.2 Literature on Asset Replacement Problem

As assets age, they generally deteriorate, resulting in rising operating and maintenance (O&M) costs and decreasing salvage values. Moreover, newer assets that have a better performance and keep better their value may exist in the marketplace and be available for replacement. Therefore, public and private organizations that maintain fleets of vehicles, and/or specialized equipment, need to decide when to replace vehicles composing their fleet. These equipment replacement decisions are usually based on a desire to minimize fleet costs and are often motivated by the state of deterioration of the asset and by technological advances (Hartman, 2005).

The general topic of equipment replacement models was first introduced in the 1950's (Bellman, 1955). By using dynamic programming, Bellman developed a model in order to obtain the optimal age of replacement of the old machine with a new machine. Another important subject was the development of parallel replacement models in which management decisions are made for a group of assets instead of one asset at the time (Hartman and Lohmann, 1997).

Vehicle replacement is a key role of fleet provisioning teams. Indeed field services operational planning and delivery primarily relies on the assumption that the whole engineering force can be furnished with the vehicle appropriate for the service, at any time. In practice, the choice of the adequate type, brand, and technology depend on internal factors (such as the engineer role, service environment and, but not systematically mileage driven), and on external factors (such as fuel price variation, government carbon emission incentives, manufacturing costs and maintenance costs). Moreover, in addition to risk and field force efficiency, the impact of vehicle replacement on customer experience needs to be considered as well. This suggests a twofold fleet planning problem that vehicle replacement aims to address: a planned fleet portfolio and a rental plan for jeopardy situations.

In addition, field services enterprises face increasing challenges on carbon emissions and cost reduction. This need to transform the way field services operate has an impact on the choice of vehicles within a business, affecting the vehicle replacement processes. When attempting to optimize the fleet composition, which is essential for achieving sustainability, we need to take into account several factors (some of which are stochastic and uncertain in nature), which need to be addressed before low-carbon vehicles are a feasible alternative for field services operations including, for example, the intangible reputation of sustainable energy investment, the evolution of market prices, strategic partnerships, and risk sharing.

2.2.1 The General Classifications of Fleet (Asset) Replacement models

The models generally can be categorized into two main groups based on different fleet (asset) characteristics: homogenous and heterogeneous models. In the homogeneous replacement models, a group of similar vehicles in terms of type and age, which form a cluster, have to be replaced simultaneously (each cluster or group cannot be decomposed into smaller clusters).

On the other hand, in the heterogeneous model, multiple heterogeneous assets, such as fleets with different types of vehicle, have to be optimized simultaneously. For instance, vehicles of the same type and with the same age may be replaced in different periods (years) because of the restricted budget for procurement of new vehicles. The heterogeneous models are closer to the real world commercial fleet replacing problem. These models are solved by Integer Programming and, generally, the input variables are assumed to be deterministic (e.g., Hartman, 1999, 2000, 2004; Simms et al., 1984; and Karabakal et al., 1994).

The most popular methodology for solving homogenous models is dynamic programming. The advantage of the homogenous model is to take into account probabilistic distributions for input variables (e.g., Hartman, 2001; Hartman and Murphy, 2006; Oakford et al., 1984; Bean et al., 1984; Bellman, 1955).

Another important classification of these models regards the nature of the replacement process: parallel vs. serial, e.g., Hartman and Lohmann (1997). The main difference between parallel replacement analysis and serial replacement analysis is that the former takes into account how any policy exercised over one particular asset affects the rest of the assets of the same fleet. An example of parallel replacement would be a fleet of trucks that service a distribution centre. In this case, the total available capacity is the sum of the individual capacities of the trucks. In the serial replacement model the assets operate in series, and consequently, demand is satisfied by the group of assets which operate in sequence. An example of this case is a production line in which multiple machines must work together to meet a demand or service constraint. In general, the capacity of the system is defined by the smallest capacity in the production line (Hartman, 2004).

The following definition of parallel replacement comes from Hartman and Lohmann (1997). Parallel replacement deals with the replacement of a multitude of economically interdependent assets which operate in parallel. The reasons for this economic interdependence are: (1) demand is generally a function of the assets as a group, such as when a fleet of assets are needed to meet a customer's demands; (2) economies of scale may exist due to purchasing assets and promoting large quantity of purchases; (3) diseconomies of scale may exist with maintenance costs because assets which are purchased together tend to fail at the same time; and (4) budgeting constraints may require that assets compete for available funds. These characteristics, either alone or together, can cause the assets to be economically interdependent.

On the other hand, the serial replacement analysis assumes a certain utilization level for an asset throughout its life cycle. Hartman (1999) mentioned that as utilization levels affect operating and maintenance costs and salvage values (which in turn influence replacement schedules) a replacement solution is not optimal unless utilization levels are also maximized. For this reason, an asset utilization level depends on the demand requirements, number of assets available, and capacity of each asset.

Next we present the different approaches to modelling the asset replacement problem: the economic life-cycle, the repair cost limit, the comprehensive cost minimization, and the issue of decreasing utilization with age.

2.2.2 Approaches for Replacement Decisions

In this section we review different approaches for deciding the optimal time for asset replacement. Throughout this section our goal is to identify replacement candidates among fleet or asset members so that the total costs are minimized in the long run.

2.2.2.1 Approaches Based on the "Economic Life"

An intuitive method for identifying replacement candidates is to use a replacement standard, such as the age of the equipment. For example, assets older than a standard threshold should be replaced. Additionally, a ranking profile can be used in order to sort the equipment units by how much they exceed the threshold. For example, Eilon et al. (1966) considered a model for the optimum replacement of fork lift trucks. The parameters in their model were the purchase price, the resale value and the maintenance costs of the equipment. The goal of their model was to derive the minimum average costs per equipment year, and the corresponding optimal equipment age policy, for a fleet of fork lift trucks.

Let us now describe the model proposed by Eilon et al. (1966) in more detail. Let TC (t) be the total average annual (or per time period) cost of an existing truck, assuming it is replaced at age (time) t. Let A stand for the acquisition cost of new truck, S (t) be the resale value of the existing truck at age t, C (t) be the accumulated depreciation costs up to time *t*, τ be the rate of taxation, and $f(t)$ be the maintenance costs of a truck, *t* years after acquisition. Then the total average annual cost of an existing truck is represented by (2.1).

$$TC(t) = \frac{1}{t}(A - S(t) - C(t)\tau) + \frac{1}{t}\int_0^t f(t)dt \tag{2.1}$$

The first term in equation (2.1) represents the average capital costs involved in the acquisition of the existing truck, taking into account the savings from resale value and tax savings from depreciation. The second term in equation (2.1) expresses the total average maintenance costs for the existing truck over the years up to the present time *t*. The minimum total average annual costs, as a function of *t,* determines the optimal replacement time.The economic life of an asset (also known as service life or lifetime of the asset) is defined as the age which minimizes the *Equivalent Annual Cost* (EAC) of owning and operating the asset. The EAC includes purchase and *Operating and Maintenance* (O&M**)** costs minus salvage values. Generally, O&M costs increase with age while salvage values decrease with age. As a result, the optimal solution represents a trade-off between the high costs of replacement (purchase minus salvage) versus increasing O&M costs over time.

The concept of economic life is easier to describe graphically. In Figure 2.1, adapted from Harman and Murphy (2006), it is assumed that the initial purchase cost is $100000, with the salvage value declining 20% per year. O&M costs are expected to increase 15% per year after $11500 in the first year. Figure 2.2 illustrates the annualized O&M and capital costs and their sum (EAC) for each possible of age assuming an annual interest rate 8% (Hartman and Murphy, 2006). Once the optimal economic life is determined, the asset should be continuously replaced at this age, if we assume repeatability and stationary costs (Hartman and Murphy, 2006).

In order to obtain the EAC, when retaining an asset for *n* periods, all costs over the *n* periods must be converted into *n* equal and economically equivalent cash flows. Then, the economic life of an asset is typically computed by calculating the EAC of retaining an asset for each of its possible service lives, ages one through *n*, and the minimum is chosen from this set (e.g., Hartman, 2005; Weissmann et al., 2003; Hartman and Murphy, 2006).

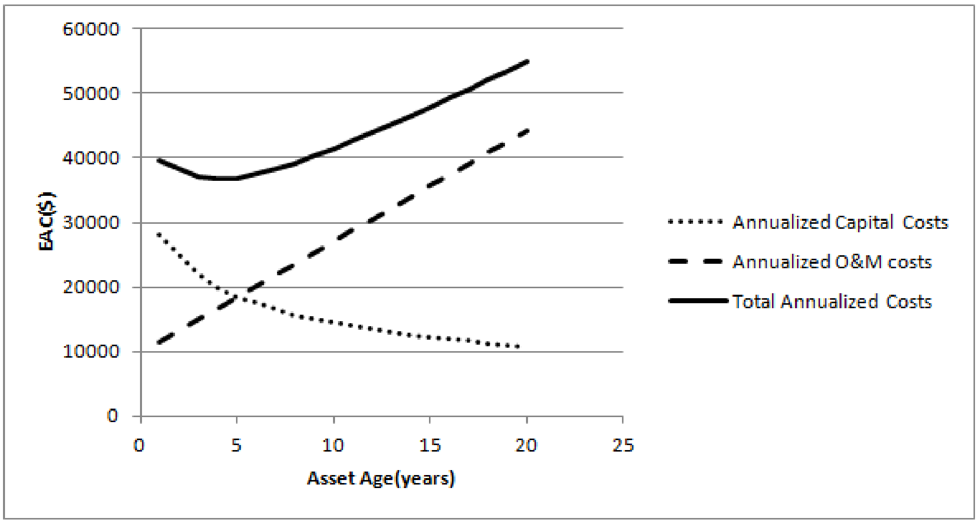

Figure 2.2: Annualized purchase cost, O&M cost, and Total (EAC) costs

Yatsenko and Hritonenko (2011) have also considered the economic life (EL) method of asset replacement taking into account the effects technologic improvements which decrease maintenance costs, new asset cost, and salvage value. They have shown that, in general, the EL method renders an optimal replacement policy when the relative rate of technological change is less than one percent. However, for larger rates, they recommend annual cost minimization over the two future replacement cycles, which was earlier proposed and implemented by Christer and Scarf (1994).

2.2.2.2 Approaches that Consider a Repair Cost Limit

Another replacement criterion is the repair cost. When a unit requires repair, it is first inspected and the repair cost is estimated. If the estimated cost exceeds a threshold, which is known as "repair limit" then the unit is not repaired but, instead, is replaced. Repair limits have long been used and their values have often been based on the principle that no more should be spent on an item than it is worth.

This criterion is indeed an important one. There is evidence that repair cost limit policies have some advantages in comparison with economic age limit policies. For example, Drinkwater and Hastings (1967) analysed data for army vehicles. They obtained the repair limiting value in which the expected future cost per vehicle-year when the failed vehicle is repaired is equal to the cost in which the failed vehicle is scrapped and a new one is substituted. Specifically, they defined two options: a) repair the vehicle and b) scrap the vehicle and replace it by a new one. This is called a repair decision. We now present the model used for the repair decision in more detail, following Drinkwater and Hastings (1967).

Consider a vehicle at age t which requires repair. If we select option *a*), to repair the vehicle, the future cost per vehicle-year is represented by (2.2) in which r is the present cost of repair, $c(t)$ is the expected total cost of future repairs, and $l(t)$ is the expected remaining life of the vehicle.

$$\frac{r+c(t)}{l(t)} \tag{2.2}$$

If we select option *b*), to scrap the vehicle, the expected future cost per vehicle-year will be δ, which is defined by the average cost per vehicle-year up to age t. Obviously, the repairing decision (option *a*) will be selected if (2.3) holds. Otherwise, the scrapping decision is chosen. Therefore, the critical value of r is determined by equation (2.4) in which the future cost per vehicle-year equals the average cost per vehicle-year up to age t. As a result, the optimal repair limit at time t, $r^*(t)$, is determined by (2.5).

$$\frac{r+c(t)}{l(t)} < \delta \tag{2.3}$$

$$\frac{r^*(t)+c(t)}{l(t)} = \delta \tag{2.4}$$

$$r^*(t) = \delta l(t) - c(t) \tag{2.5}$$

Drinkwater and Hastings (1967) have shown that the repair cost limit policy is better than the economic age policy. Nonetheless, there is a main drawback to the conventional repair cost limit policy: the repair/replace decision is based only on the cost of one single repair. Under this condition, a system with frequent failures and, consequently, high accumulated repair costs will continue to be repaired rather than replaced. As a result, an improved policy making the repair/replace decision based on the entire repair history would be a better criterion.

In order to address this issue, Chang et al. (2010) have developed a generalized model for determining the optimal replacement policy based on multiple factors, such as the number of minimal repairs before replacement and the cumulative repair cost limit. The main characteristic of their model is to consider the entire repair-cost history. Nakagawa and Osaki (1974) have also suggested an alternative approach which does not focus on repair costs but, instead, on repair time. If the repair process is not completed up to the fixed repair time limit, then the unit under repair is replaced by a new one. The repair time limit is obtained by minimizing expected costs per unit of time over an infinite time horizon.

2.2.2.3 Comprehensive Cost Minimization Models

There are other approaches that generalize the problem of optimal replacement by taking into account the optimal decisions for acquisition, operation, and replacement policies. For example, Simms et al. (1984) have analysed a transit bus fleet in which the equipment units in the fleet system were assigned to perform different tasks, at different levels, subject to changing capacity constraints. Their objective was to minimize the total discounted cost over a finite horizon.

The objective function is represented by (2.6), in which t and a are the indices for time periods (year) and age of the buses, respectively, and T is the length of the planning horizon, in years. The decision variables are: the number of route kilometres travelled by a bus with age a, in year t, m_{ta}; the number of buses with age a, which operate in year t, x_{ta}; and the number of new buses which should be purchased, with an acquisition cost L_t, at the beginning of year t, denoted by p_t. In each year the price of selling a bus with age a, is represented by S_{ta} , and $C_{ta}(m_{ta})$ is the cost of operating a bus with age a, in year t, for the associated kilometres travelled by m_{ta}. Finally, γ represents the discount factor. In equation (2.6) the first term represents the acquisition costs, the second term stands for the revenue received from selling the buses, and the third term denotes the cost of operating the buses. Simms et al. (1984) computed the optimal acquisition, operation and selling policies using dynamic programming.

$$\underset{m_{ta},x_{ta},p_t}{Min}\ Z=\sum_{t=0}^{T}\gamma^t p_t L_t-\sum_{t=0}^{T}\gamma^{t+1}\sum_{a}(x_{ta}-x_{t+1,a+1})S_{t+1,a+1}+\sum_{t=0}^{T}\sum_{a}\gamma^i x_{ta}C_{ta}(m_{ta}) \qquad (2.6)$$

Next we consider the constraints of the model proposed by Simms et al. (1984). The nonlinear constraint (2.7) requires that a minimum total route kilometers, per year, M_t, is driven by the fleet. The constraint (2.8) expresses the boundary conditions for the decision variable m_{ta}, in which m_- and m_+ denote the minimum and maximum number of kilometres that a single bus can drive in a given year, respectively. Constraint (2.9) represents the requirement that at least a minimum number of buses, N_t, in each year, should be in the fleet. In inequality (2.10), Q is the minimum age for a bus to be considered for a sell decision and the left hand side is equal to the number of buses which are sold at the beginning of the corresponding year. Therefore, we can say that inequality (2.10) is a consistency constraint, in the sense that it does not permit old buses to be bought. Equation (2.11) means that the buses are not eligible for sale until their reach to the minimum age Q. Equation (2.12) represents the boundary conditions, in which K_a are the initial numbers of buses for the different ages. If budget constraints for capital acquisitions are also considered, then the constraint (2.13) is also required, in which B_t is the capital budget in period t. Furthermore, if there is also an operating budget constraint, then we also need to impose constraint (2.14) in which O_t is the operating budget, in period t.

$$\sum_a x_{ta} m_{ta} \geq M_t \qquad \forall\ t \in \{0,1,2,\ldots,T\} \tag{2.7}$$

$$m_- \leq m_{ta} \leq m_+ \tag{2.8}$$

$$\sum_a x_{ta} \geq N_t \qquad \forall\ t \in \{0,1,2,\ldots,T\} \tag{2.9}$$

$$x_{ta} - x_{t+1,a+1} \geq 0 \quad , a \geq Q-1 \tag{2.10}$$

$$x_{ta} - x_{t+1,a+1} = 0 \quad , a < Q-1 \tag{2.11}$$

$$x_{(-1)a} = K_a \quad , x_{(T+1)j} = 0 \tag{2.12}$$

$$p_t \leq \frac{B_t}{L_t} \tag{2.13}$$

$$\sum_a x_{ta} C_{ta}(m_{ta}) \leq O_t \tag{2.14}$$

The model represented by equations (2.6)-(2.14) has a non-linear objective function subject to a set of non-linear constraints. By using dynamic programming, Simms et al. (1984) solved it. If we compare the two models proposed by Simms et al. (1984) and Keles and Hartman (2004), we understand that regardless of the methodology for solving two models, the main difference is considering the behaviour of utilization as a function of age of the vehicles and assuming it as a decision variable by Simms et al. (1984). Another difference is that Simms et al. (1984) considered the same type of asset whereas Keles and Harman (2004) considered multiple types of asset. However, for the rest of the components of the two models, i.e., the goal of the objective function and the constraints they are almost the same.

On this same topic, Hartman (1999) has considered the replacement plan and corresponding utilization levels for a multi-asset case in order to minimize the total cost. He generalized equipment replacement analysis as it explicitly considers utilization as a decision variable. His model allows assets to be categorized according to age and cumulative utilization, while allowing their periodic utilization to be determined through analysis. As a result, he has considered simultaneously tactical replacement and operational decisions, taking into account the tradeoffs between capital expenses (replacement costs) and operating expenses (utilization costs). The objective was to minimize the total cost of assets that operate in parallel. He solved the problem using linear programming. Furthermore, Hartman (2004) has generalized this same problem by incorporating a stochastic demand. He solved the problem using dynamic programming. Overall, both Simms and Hartman did not introduce any special new replacement criteria and just presented optimization methodologies in order to minimize the cost of corresponding fleets.

Furthermore, an important issue that we need to discuss in this topic is the relation between age and utilization. The utilization intensity (annual mileage) of vehicles exploited by transportation companies decreases with time of exploitation/cumulative mileage probably in all real life cases. The youngest vehicles are usually utilized more intensively than the oldest ones, because their unit exploitation costs are lower (e.g. fuel consumption is lower), and the depreciation costs can be ignored. The occurrence of such pattern can be found in, for example, Kim et al. (2004) and Simms et al. (1984), and it fits well with real world situations, as illustrated in Figure 2.3 (based on Simms et al., 1984).

Simms et al. (1984) have considered explicitly this issue in a bus fleet data. They mentioned that if the relation between utilization with age is not considered, one would expect that the older buses would be replaced first and younger buses kept. However, in practice, this is not the case for two reasons. First, the case in which older buses are kept only to meet peak daily demand and these buses accumulate only the minimum number of route kilometres during the year. Second, the resale value of younger buses is much higher than older buses. Therefore, even if the operating cost of older buses is higher, they do not operate enough route kilometres and the extra expense is lower than the gain obtained by selling younger buses. So, they assumed two levels of utilization for an urban transit bus fleet with different ages. They concluded that a high utilization level is considered for buses with less than ten years for satisfying the normal demand and a low utilization level for buses more than ten years in the case of peak demand.

Redmer (2009) has also considered the relationship between utilization intensity and aging by applying the minimal average cost replacement policy using the following considerations. a) The utilization intensity (annual mileage) of vehicles for each year of their operational life has to be taken into account. b) The vehicles' exploitation costs have to be divided into fixed costs (independent of utilization intensity, but varying with time of exploitation/cumulative mileage), running costs (depending on utilization intensity/mileage and varying with time of exploitation/cumulative mileage) and fuel costs (varying with time of exploitation/cumulative mileage). c) The total costs of exploitation and ownership have to be given per one km or mile. d) The technical durability of vehicles (e.g., maximal mileage) has to be taken into account. e) Different forms of financing the fleet investments (buying for cash, credit, leasing, and hiring) have to be considered.

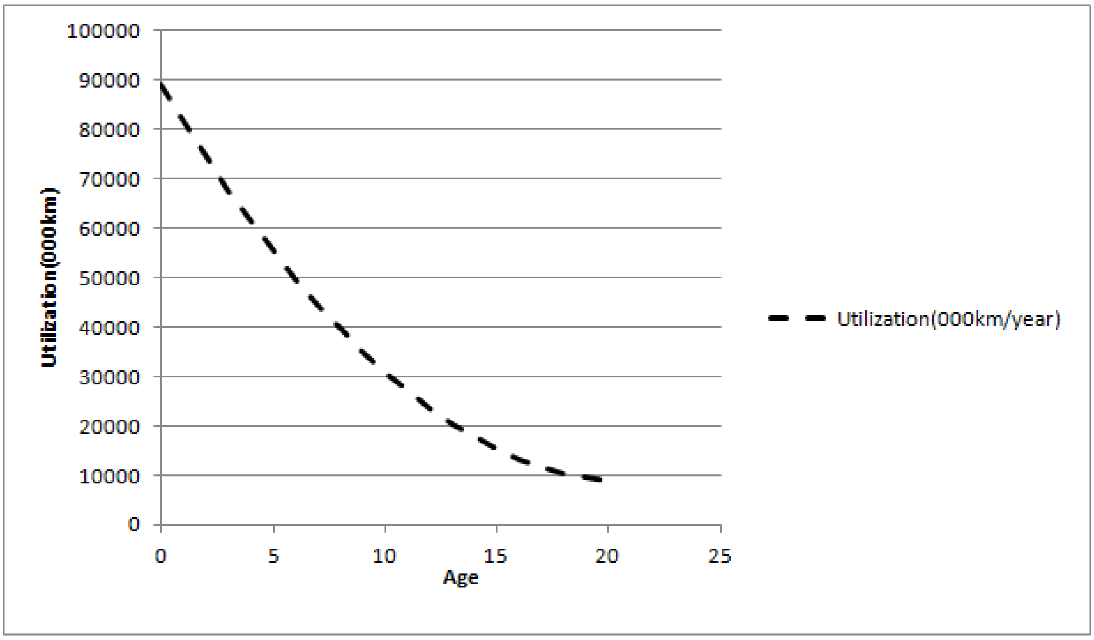

Figure 2.3: Annual utilization by age.

Next, we describe the parallel replacement problem and we suggest a new model for addressing the issues raised by Redmer (2009) in the context of parallel replacement problem (Section 2.4).

2.2.3 The General Parallel Replacement Problem

In this section we commonly refer to groups of assets as fleets. However, the model is general in the sense that cost functions are specified without operational details. Thus, this analysis may be applied to a manufacturing setting if the costs can be quantified.

The parallel replacement models are usually difficult to solve due to their combinatorial nature as mentioned by Hartman (2000). Jones et al. (1991) considered a parallel replacement problem on the condition of fixed replacement costs. Rajagopalan (1998) and Chand et al. (2000) have proposed dynamic programming algorithms that simultaneously consider the replacement and capacity expansion problems.

2.2.3.1 An Integer Programming Formulation of the Parallel Replacement Problem

Given the complex nature of the problem, the case of multiple alternatives within parallel replacement has been rarely considered in the literature. Keles and Hartman (2004) have proposed an Integer Programming formulation of the bus fleet replacement problem with multiple choices under economies of scale and budgeting constraints. The objective function is summarized in equation (2.15). All costs in the model are assumed to be discounted to time zero using an appropriate discount rate. The fixed cost associated with asset buying is represented by f_t and l_{it} is the new asset acquisition cost per unit in each year. The operating and maintenance cost is shown by c_{iat} and the salvage revenue is represented by r_{iat}.

In (2.15) the indices are a, t, and i which stand for the age of the assets (buses), time periods, and type of the assets, respectively. I, represents the total number of challengers (i.e., available alternatives for assets) in each period. The maximum age of any asset associated with its type is shown by A_i and the length of time horizon is assumed to be T (typically T is assumed to be less than 15 years). The total number of assets which are currently used in the system is represented by X_{iat} ($a > 0$). The decision variables are the number of the assets bought at the beginning of each year, X_{i0t}, the number of assets which are salvaged at the end of each year, S_{iat}, and a binary variable confirming an acquisition in year t, Z_t.

$$\underset{X,S,Z}{Min} \sum_{i=1}^{I} \left[\sum_{t=0}^{T-1} \left(f_t Z_t + \sum_{a=0}^{A_i-1} l_{it} X_{i0t} \right) + \sum_{t=0}^{T-1} \sum_{a=0}^{A_i-1} c_{iat} X_{iat} - \sum_{t=0}^{T-1} \sum_{a=1}^{A_i} r_{iat} S_{iat} \right] \qquad (2.15)$$

The objective function represents the costs associated with each challenger's discounted cash flows which are purchasing, operating and maintenance costs subtracting the revenue from salvage values.

We now describe the constraints in the Keles and Hartman (2004)'s model. Constraint (2.16) requires that enough assets (or capacity) are available to satisfy demand for buses at time t, d_t. Equation (2.17) presents the capital budging constraint to limit the payment for new asset acquisitions with predetermined capital budget, b_t, in each year. Constraint (2.18) describes that the initial number of assets, h_{ia} ($a > 0$), should be either used, X_{ia0}, or salvaged, S_{ia0}. Equation (2.19) shows that the number of used assets in one year should be either used or salvaged in the next year. Constraint (2.20) requires that all assets should be sold in the last year of the planning horizon (T). Equation (2.21) presents that any asset that has reached its maximal age is not used anymore. Constraint (2.22) prohibits salvaging any new asset immediately. Indeed, for salvaging of any new purchased asset at least one year should be passed. Finally, constraint (2.23) requires non-negative, integer solutions.

$$\sum_{i=1}^{I}\sum_{a=0}^{A_i-1} X_{iat} \geq d_t \qquad \forall\ t \in \{0,1,...,T-1\} \tag{2.16}$$

$$\sum_{i=1}^{I}\sum_{a=0}^{A_k-1} l_{it} X_{i0t} + f_t Z_t \leq b_t \qquad \forall\ t \in \{0,1,...,T-1\} \tag{2.17}$$

$$X_{ia0} + S_{ia0} = h_{ia} \qquad \forall\ \ a \in \{1,2,...,A_k\}, \forall i \in I \tag{2.18}$$

$$X_{i(a-1)(t-1)} = X_{iat} + S_{iat} \quad \forall\ i \in I, \forall a \in A_i, \forall t \in \{1,2,...,T\} \tag{2.19}$$

$$X_{iaT} = 0 \qquad \forall\ a \in \{0,1,2,...,A_i - 1\} \tag{2.20}$$

$$X_{iA_it} = 0 \quad \forall\ i \in I, \forall t \in \{0,1,2,...,T\} \tag{2.21}$$

$$S_{i0t} = 0 \qquad \forall i \in I, \forall t \in \{0,1,2,...,T\} \tag{2.22}$$

$$X_{iat}, S_{iat} \in \{0,1,2,...\},\ Z_j \in \{0,1\} \tag{2.23}$$

Keles and Hartman (2004), by solving the model represented in equations (2.15)-(2.23), together with an extensive sensitivity analysis, have considered the impact of various parameters on the optimal policies for choosing the appropriate type and timing for bus replacement.

The aforementioned papers on the parallel replacement problem were considered in a deterministic framework. Replacement models in the case of existence of uncertainty were focused mainly on single or serial replacement problems. For example, Ye (1990) presented a single replacement model in which operating costs and the rate of deterioration of equipment were stochastic and the optimal time for replacing was determined in a continuous-time setting. Dobbs (2004) developed a serial replacement model in which operating costs were modelled as a geometric Brownian motion and the optimal investment time was obtained. Rajagopalan et al. (1998) developed a dynamic programming algorithm for a problem where sequences of technological breakthroughs were anticipated but their magnitude and timing were uncertain. A firm, operating in such an environment, should decide how much capacity of the current technology to acquire to meet future demand growth.

Keles and Hartman (2004)'s model has been very successful in other types of applications. For example, Feng and Figliozzi (2013) have considered a fleet replacement framework for comparing the competitiveness of electrical with conventional diesel trucks. Their model has been adapted from Keles and Hartman (2004). They obtained scenarios with different fleet utilization and fuel efficiency. By using sensitivity analysis of ten additional factors, they have shown that EVs are more cost effective when conventional diesel vehicles' fuel efficiency is low and daily utilization is above some threshold. Breakeven values of some key economic and technological factors that separate the competitiveness between EVs and conventional diesel vehicles were calculated in all scenarios.

Typically, in the comparison of the performance of electrical and conventional vehicles, one takes into account the high capital costs associated with electrical engine vehicles. The replacement decision depends on the result of a complete economic and logistics evaluation of the competitiveness of the new vehicle type. In addition, as vehicles age, their per-mile operating and maintenance costs increase and their salvage values decrease. So, when the O&M costs reach a relatively high level, it may become cost effective to replace fossil fuel vehicles since the savings from O&M costs may compensate the high capital cost of purchasing electrical engine vehicles. Moreover, if fleet managers are enthusiastic in replacing conventional vehicles with new electric vehicles, it is important to understand how the O&M costs and salvage values change over time. Conventional diesel and electric commercial vehicles have significantly different capital and O&M costs.

2.2.3.2 A General Parallel Heterogeneous Asset Leasing Replacement Model

In this subsection we introduce a general asset (fleet) replacement model for obtaining optimal replacement decisions regarding K types of assets under leasing framework. This model is adapted from Keles and Hartman (2004). Specifically, a heterogeneous model is developed in which the assets are bounded by common budget constraints, demand constraints, and a fixed cost that is charged in any period in which there exist a replacement. It is assumed that in any period, assets from any of K types can be leased in order to replace retired assets for meeting corresponding demand in that period. First, we introduce the general asset replacement model and then we consider the customized model for fleet replacement.

The notation and formulation to be presented is more easily described by the network in Figure 2.4. For the sake of simplicity this figure represents the case of two asset types that are available to meet demand (i=2). The age of the asset, a, is defined on the y-axis (maximum A) and the end of the time period, t, is defined on the x-axis (horizon T). Due to the fact that we are considering a commercial setting, the leasing period is assumed to be four years. So, based on this assumption in Figure 2.3, the model is represented with $A = 3$ and $T = 6$. Indeed, at the end of time horizon $T = 6$ all the assets are retired.

Each node is defined according to the pair (a,t) and flow between these nodes, X_{iat} represents an asset of age a in use from the end of time period t to the end of period t +1, in which the asset is of age a+1. Assets are either provided from the initial fleet, represented as flow from supply nodes n_{ia}, or must be leased, represented as X_{i0t} flow in each period t.

An asset when reaches age A must be retired. All assets are retired at the end of the horizon. For meeting the associated demand in each period, the retired assets should be replaced by leasing new assets. In Figure 2.4, the two types of assets are represented by different arcs (dashed or solid).

Next we adapt the introduced model for fleet replacement. We consider two types of technologies: the fossil fuel technology (Defender) and the new engine technology (Challenger). Moreover, we take into account the leasing option for financing the commercial fleet investments which is the best option in the commercial setting (Redmer, 2009). This is a deterministic model. Future costs such as lease prices, fuel prices, fuel and electricity consumption rates and many other economic and technical factors are assumed to be known functions of time and vehicle type.

The indices in the model are the types of vehicles, $i \epsilon \{1, 2\}$, the maximum age of vehicles in years, a $\epsilon A = \{1, 2,..., A\}$, and the time periods (year), $t \epsilon T = \{0, 1,..., T\}$. The decision variables include the number of type i, age a vehicles which are currently leased in year t, X_{iat}, and the number of type i vehicles which are leased at the beginning of year t, P_{it}.

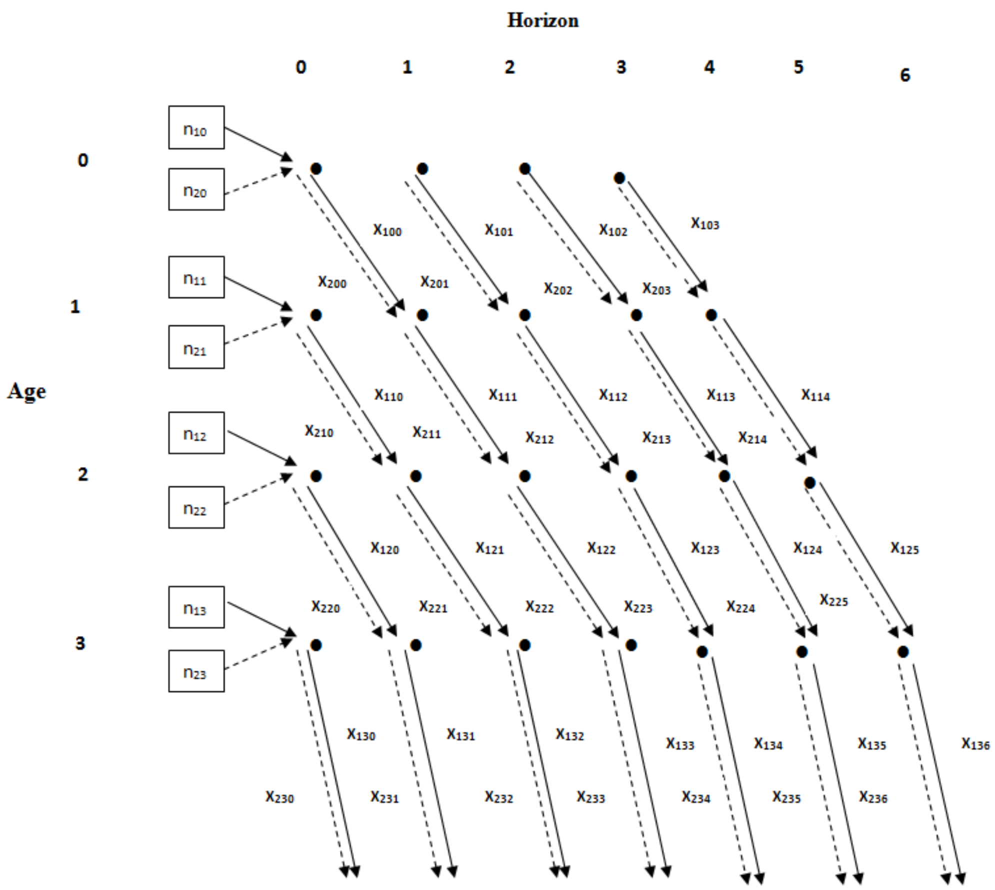

Figure 2.4: Challengers are denoted by different arcs and different source (initial fleet) nodes. Nodes are labelled (*a, t*) with *a* the age of the asset and *t* the time period. Flow X_{iat} represents asset leased (a = 0) and assets in use (a > 0).

The parameters are a) the expected utilization (miles travelled per year) of a type *i*, age *a* vehicle in year *t* (miles/year), u_{iat}; b) the expected demand (miles need to be travelled by all vehicles) in year *t* (miles), d_t ; c) the available budget (money available for leasing new vehicles) in the beginning of year *t*, b_t ; d) the initial number of a type *i*, age *a* vehicles at the beginning of first year, h_{ia} ; e) the lease cost of a type *i* vehicle, l_i ; f) the expected per mile operating (running) cost of a type *i* , age *a* vehicle in year *t*, o_{iat} ; and g) per mile emissions cost of a type *i* , age *a* vehicle, e_{ia}. The objective function which we want to minimize (2.24) is the sum of leasing costs for the period (*T*-3) and the operating (running) cost for the entire horizon to the end of year T. Moreover, equation (2.25) shows that the leasing costs cannot exceed the yearly budget and equation (2.26) requires that the total miles travelled by all used vehicles meet the yearly demand. Equation (2.27) describes the total number of the vehicles with different ages and types in the first year should be equal to the initial condition of the system. In addition, equation (2.28) shows that in the last 4 years of the planning horizon there is no leasing of new cars. In equation (2.29) the number of new leased cars at the beginning of each year is determined. Equation (2.30) represents the flow equation in which the number of the cars at each year equals to the number of new leased cars plus the number of cars belonged to the previous year. Finally, expression (2.31) is the constraint for non-negative numbers of decision variables.

$$Min \sum_{i=0}^{I}\sum_{t=0}^{T-3}(l_i P_{it}) + \sum_{i=0}^{I}\sum_{a=0}^{A}\sum_{t=0}^{T}[o_{iat} + e_{ia}]u_{iat}X_{iat} \tag{2.24}$$

$$\sum_{i=i}^{I} l_i.P_{it} \le b_t \qquad \forall t \in \{0,1,2,...,T-3\} \tag{2.25}$$

$$\sum_{a=0}^{A}\sum_{i=}^{I} X_{iat}u_{iat} \ge d_t \quad \forall t \in \{0,1,2,..,T-3\} \tag{2.26}$$

$$X_{ia0} = h_{ia} \qquad \forall i \in I, \forall a \in A \tag{2.27}$$

$$P_{it} = 0 \qquad \forall i \in I, \forall t \in \{T-3,...,T\} \tag{2.28}$$

$$P_{it} = X_{i0t} \quad \forall\ i \in I, \forall t \in \{0,1,2,...,T-3\} \tag{2.29}$$

$$X_{iat} = P_{it} + X_{i(a-1)(t-1)} \quad \forall\ i \in I, \forall a \in A, \forall t \in T \tag{2.30}$$

$$X_{iat}, P_{it} \in Z^{+} \tag{2.31}$$

Having analyzed extensively the different models in the literature and identified some of their limitations, next, in Section 2.4, we summarize the main insights from our review of these different approaches.

2.2.4 Insights from the Literature on Fleet (asset) Replacement Models

The aforementioned replacement policies and methods represent only a small part of all efforts that have been done to solve the equipment replacement problem in general (Nakagawa, 1984; Ritchken and Wilson, 1990), and the vehicle replacement problem in particular (Eilon et al., 1966).

Despite the fact that the vehicle replacement policy has a prominent role in transportation companies and belongs to an important class of the fleet strategic management problems that have been extensively considered in the literature during last 50 years (Dejax and Crainic, 1987), there are many obstacles for applying the existing methods. Such obstacles exist from the following features of the existing replacement methods (Redmer, 2009):

- Most of the methods are assumed to be applied in a stable environment which is not the case for most of the vehicles in under operational conditions. For example, the way those vehicles are utilized and the loads carried, the climate, and other factors from road conditions which can have impact on fuel economy of the vehicles.
- Focused on a given group (type) of vehicles instead of a single vehicle.
- Taking into account a constant utilization rate of the equipment during its Operational life.

In Practice, the existing models have at least one of the mentioned drawbacks. For instance, Eilon et al. (1966) consider particular vehicles but assume a fixed utilization pattern whereas Simms et al. (1984) relax the assumption of the constant utilization but constrain an age to the replacement problem by placing a lower bound of 15 years. Suzuki and Pautsch (2005) also constrain an age to the replacement model by putting an upper bound of 5 years and they conclude that vehicles of age 6 or beyond may not be suitable for business operations, that contradicts the assumption of Simms et al. (1984). Moreover, the significant part of the vehicle replacement models assumes budget constraints (Simms et al., 1984), which is important when replacement policy is defined for fleet of vehicles but not for particular vehicles. However, such constraints generally result in replacement of the limited group of the oldest vehicles (Redmer, 2009). Because of the listed drawbacks of the existing replacement methods, a direct application of them to the vehicles deployed by freight transportation companies is difficult, if not impossible.

2.2.5 Practical Challenges for the Fleet Replacement Problem

Typically fleet management for field services requires finding the right vehicle, of the right capacity, for the right business, and fitting the required features into the serviced work type. In practice, these decisions are twofold:

- First, we need to identify the vehicles portfolio needs in terms of volume capacity, driving features (speed and driving wheels, for instance).
- Second, we need to calculate a replacement plan, from one to five years, to ensure that the provision of the right brand, model, and vehicle asset supplier for each identified fleet item.

The second step can be modeled as a multi-objective combinatorial optimization problem. However there is not a single solution, as a matter of fact, the solution is in the form of a ranking of the technology and brands available based on the most economical and ecological choice. The accuracy of such a ranking is generally limited to a number of years; due to high variation in energy prices market, fleet managers generally are advised to plan one year in advance. Therefore, there is an important practical challenge: to increase the planning horizon to the full four years, taking into account all the uncertainties.

The combinatorial aspect of the operation is complicated by the fact that the matching of vehicle types and running technology depends both on the driver's behavior and on the variation of usage over days, months or years. For instance, a simple analysis suggests that the petrol engine tends to be cost effective when dealing with short annual mileage usage, and a mixed diesel and hybrid technology are suitable for normal distances while affording a risk exposure reduction.

Moreover, the electric engine tends to be the optimal choice, both from risk and cost minimization perspectives, when the annual mileage usage is high.

The following are some of the challenges faced by fleet provisioning:

- The fleet provisioning needs to consider the mileage driven by the vehicles. Thus, in the process of constructing a replacement tactical plan, we need to implement a method for forecasting annual mileage with a granularity at the vehicle type or service operations type level.

- The length of equipment life is not fixed. Even though the rental duration can be used as working hypothesis, in practice the replacement decision may happen before the planned end of life, depending on the maintenance cost, fuel prices variation forecast, electric energy recharge constraints, geography and volume of the field service demand.

- Overall, we need to find a balance between risk exposure and O&M cost minimization, taking into consideration the utilization of vehicles, and the frequency of long, medium or short distance driven by each vehicle. A fine granularity analysis of mileage, fuel consumption and geographical information monitoring data will help in adjusting the approach for realizing sustainable field operations.

- There is a need to consider fuel price uncertainty, the variation of fuel consumption in each technology, leasing costs and the accessibility of vehicles based on the real data.

- If we consider a larger number of aspects in the model the analysis will be more accurate. If you want to introduce manufacturing costs into the model you will require quote information from the enterprise processes; if you consider customer experience (service commitment delivered, number of visits before completing the task, asset missing, for instance) you will need to analyze the robustness of the replacement plan when environment or service engineering variables change. Furthermore, an analysis of the impact of the average speed of the vehicles on the fleet management decisions seems to be one other direction of research; however this variable suffers generally from data quality issues, due to lack of links between tactical planning and the travel feedback from field workers: the use of an electronic box embedded in vehicles is an interesting alternative to improve the flow of information from operations to strategic planning, one of which should be considered if the improvements in fleet management outweigh the costs of installing and maintaining the system.

Additionally, the vehicle utilization governance within a firm also has an important impact on fleet management. We can consider this issue if we analyze the fleet portfolio life cycle at an organizational level. In this framework, a vehicle is seen as an item that can be swapped across business units: in this case, the transfer of an unused vehicle from a line of business to another one would be a better alternative to rent a new vehicle. If we consider this new framework several questions arise: which option leads to the best cost risk and customer experience trade-off? How can the cost of vehicle reuse option be recorded?

This governance structure at a global level, when transforming the fleet portfolio and the impact on environment, requires support at a tactical level, by: 1) planning the number of vehicles per technology (source of energy), capacity and various mileages, in the short, medium and long-term; 2) analysing risk exposure (taking into account the forecasted demand and supply life cycle); 3) considering the impact of such decisions on the customer experience.

In order to have a better grasp of the different kind of uncertainties in the fleet management, we provide a brief literature on risk management issues namely differtent kind of risks and risk measures in the next section.

2.3 Literature on Risk Management in Operations and Supply Chains

Risk and uncertainty has always been an important issue in operations and supply chain management. In the field of supply chain management, several publications have addressed the question of how to define supply chain risk. Two different approaches can be distinguished (1) risk as both danger and opportunity and (2) risk as purely danger. The first approach is in the line with search such as finance. Here the fluctuations around the expected value (mean) of a performance measure are used as proxy for risk, where is equated with variance and covers both a "downside" and an "upside" potential. That is, to say that risk is essentially a manifestation of uncontrollability rather than merely a downside possibility (Arrow, 1970).

However, the second approach which defines risk as purely danger coincides with a majority of business researches. For example, March and Shapira (1987) empirically examine how managers perceive risk and react to it. They find that the majority tend to overrate the "downside" potential of risk. Several scholars in the supply chain management field have adopted this view. Harland et al. (2003); for instance, discuss several definitions and conclude that supply chain risk is associated with the ”chance of danger, damage, loss, injury or any other undesired consequences.”

Juttner et al. (2003) have argued that supply chain risk management consists of four key elements (1) assessing the risk sources for the supply chains, (2) defining the supply chain adverse consequences, (3) identifying the risk drivers, and (4) mitigating risks for the supply chain.

There exist various types of supply chain risks. Chopra and Sodhi (2004) categorized supply chain risks into disruptions, delays, systems, forecast, intellectual property, procurement, receivables, inventory, and capacity. For example, disruption risks include natural disaster, labor dispute, supplier bankruptcy, war and terrorism, and dependency on a single source of supply as well as the capacity and responsiveness of alternative suppliers.

Kleindorfer and Sadd (2005) have provided a conceptual framework that explains the joint activities of risk assessment and risk mitigation that are fundamental to risk management in supply chains. Their assumption is that a company is interested in the tradeoff between the cost of risk mitigation investments, including the cost of management systems, and the expected costs of risks.

In the next section we focus on the risk assessment which means the quantification of the risk by introducing different kind of risk measures which are used by risk managers.

2.3.1 Risk Measuring

Quaranta et al. (2008) mention that the problem related to variance as a risk measure is that it takes into account the upside and downside of distributions equally. As a result, financial specialists focused on quantile based measures, like value at risk (VaR). The definition of VaR (e.g., Anderson, 2014) is the maximum potential loss that a financial sector can tolerate with a certain likelihood during a finite period.

However, VaR, if considered in the framework of coherent risk measures, lacks subadditivity and, consequently, convexity (Artzner et al., 1997) for general distributions (although it may be subadditive for special cases, e.g., for normal distributions). To solve these problems, recent literature has focused on coherent risk measures such as CVaR, e.g., Rockafellar and Uryasev (2000, 2002). Moreover, the cases of asymmetric asset distributions (Goh et al., 2012) and a robust optimization approach for CVaR (Chen et al., 2010) have also been considered in the literature. Next, we consider each of the above definitions in more detail.

2.3.1.1 Coherent risk measure

After 1997, with the emergence of Thinking Coherently (Artzner et al., 1997), certain conditions in which a statistic should have in order to be supposed a coherent risk measure was defined. Artzner et al. (1997) defined four conditions that have to be satisfied by a coherent risk measure. If X and Y show portfolio returns, $\rho(X)$ and $\rho(Y)$ are their risk measures, respectively, and h is a constant then we should have:

Translation Invariance: $\rho(X+h)=\rho(X)+h$ (2.32)

Positive Homogeneity: $\rho(hX) = h\rho(X)$ (2.33)

Subaddivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$ (2.34)

The above conditions which supposed to be satisfied for a coherent risk measure in summary denote that if a portfolio is riskier than another portfolio, it will have a higher risk value as long as the risk measure is coherent. In contrast, if a risk measure does not satisfy all three mentioned conditions might give a wrong evaluation of corresponding risks (Acerbi and Tasche, 2002).

2.3.1.2 Value at risk

In recent years taking into account the impacts of unexpected losses which influence on financial markets has considerably increased. After the late '80s, the Basel Committee (Basel Council, 1996) has focused on finding a better way for measuring the risk and introduced some mathematical and statistical theories for its quantification. This is the foundation that tells us the selection of VaR as risk measure. In Section 2.3.1, we mentioned a brief description of VaR. But, if we want to put it in more detail and define it more accurately, we should say that VaR concentrate on the downside risk of a portfolio and can be expressed as the maximum expected loss at a certain confidence level say 99% over a finite time horizon like twenty days. For instance, if VaR is \$-150 for a portfolio with confidence level of 99% and a time horizon of 20 days, we can say that with 99% likelihood we will not lose more than \$150 over the next twenty days.

The mathematical definition of VaR is:

$$F(\beta) = \int_{-\infty}^{VaR_\beta} p(r)dr = P\left[r \le VaR_\beta\right] = 1 - \beta \qquad (2.35)$$

In above formula $p(r)$ is the probability density function and expected rate of return, *r,* is a random variable defined by its cumulative distribution function *F*. Moreover, β is the confidence level.

VaR satisfies the conditions (2.32) to (2.33), (Artzner et al., 1999), but does not fulfill the subadditive condition and this prohibits it to be a convex risk measure. Moreover, this risk measure is very hard to optimize even when it is calculated using scenarios; in this case VaR is non-convex, non-smooth and has multiple local extrema (Uryasev, 2000).

In order to solve this problem Rockafellar and Uryasev (2000, 2002) introduced another risk measure known as conditional value at risk (CVaR).

2.3.1.3 Conditional value at risk

Another powerful risk measure, with more sound characteristics, is CVaR, which is also named Mean Excess Loss, Mean shortfall, or Tail VaR (Uryasev, 2000). Briefly, CVaR is defined as the conditional expectation of the losses beyond VaR (e.g., Anderson, 2014). CVaR was created to be an extension of VaR. The VaR model does allow managers to limit the probability of incurring losses caused by certain types of risk - but not all risks. In an intuitive way, the problem with relying only on the VaR model is that the scope of risk assessed is limited, since the tail end of the distribution of loss is not typically assessed. Therefore, if losses are happened, the amount of the losses will be substantial in value. However, CVaR gives us information about the magnitude of the losses beyond VaR. Mathematically speaking, CVaR is derived by taking a weighted average between the value at risk and losses exceeding the value at risk or:

$$CVaR_\beta = \frac{1}{1-\beta} \int_{-\infty}^{VaR_\beta} rp(r)dr \tag{2.36}$$

Or equivalently

$$CVaR_\beta = E[x \mid x \geq VaR_\beta] \tag{2.37}$$

In order to have a better grasp of the concepts of VaR and CVaR as risk measures, we have depicted them in Figure 2.4. This figure shows that VaR is a quantile which shows the greatest loss with confidence level 1-β and CVaR is the average of the losses that are exceeding the VaR and less than maximum portfolio loss with the same confidence level. Indeed, in contrast to VaR, CVaR provides extra information on the losses in the tail of the loss distribution beyond VaR (Figure 2.4).

CVaR is a consistent measure of risk because it is subadditive and convex (Artzner et al., 1999). Moreover, it has been proven that it can be optimized using linear programming, which can handle portfolios with a very large number of scenarios (Rockafellar and Uryasev, 2000). In addition, minimization of CVaR leads to near optimal solutions for VaR, and when the return-loss distribution is normal, these two risk measures produce the same optimal portfolio (Rockafellar and Uryasev, 2000). The linear program model suggested by Rockafellar and Uryasev for simultaneous minimization of CVaR and calculation of VaR is as follows:

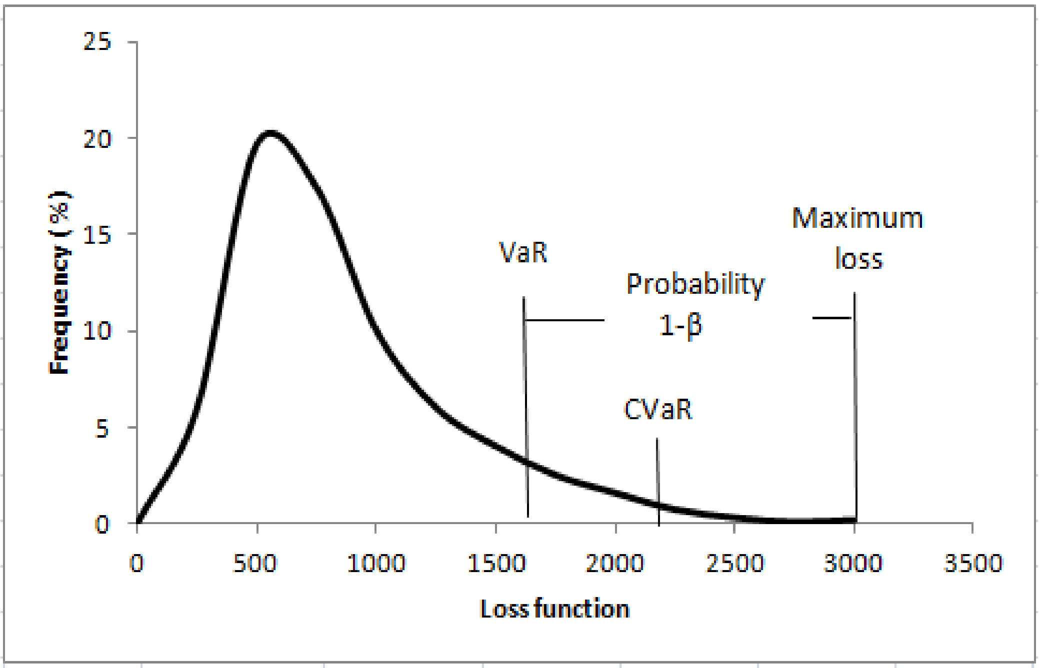

Figure 2.5 VaR, CVaR, and Maximum loss, Rockafellar and Uryasev, 2000

$$\min_{x \in X, z \in R, \alpha_\beta \in R} \phi_\beta = \min_{x \in X, z \in R, \alpha_\beta \in R} \alpha_\beta + v \sum_{q=1}^{Q} z_q$$

$$s.t.\ x \in \mathrm{X},$$

$$\mathrm{z}_q \geq f(x, \gamma_q) - \alpha_\beta, \quad \mathrm{z}_q \geq 0, \quad q = 1, \ldots Q$$

In the above model, ϕ_β and α_β denote the CVaR and VaR for the confidence level of β, respectively. In addition, q represents the number of scenarios, and γ_q shows the vector of stochastic variables in scenario q sampled from the distribution of the stochastic processes in the model, $v = ((1-\beta)\ Q)^{-1}$, where x is the vector of decision variables, z_q are positive dummy variables, and f denotes the loss function. Solving the above LP model simultaneously yields the optimal value of ϕ^*_β, the decision variable, which is x^*, and α^*_β.

2.4 Summary

In this Chapter we have provided a literature review for the three interconnected parts of this thesis which are sustainable operations, asset (fleet) replacement problem, and risk management in operations.

As we have seen, the major part of the literature in sustainable operations concentrates on green and sustainable supply chain, closed-loop supply chains, reverse logistics, and remanufacturing (Debo et al., 2005; Flapper et al., 2005; Savaskan et al., 2004). This thesis can be indirectly related to green supply chains in the sense that it focuses on justifying adoption of a green product (i.e., EVs) and the impact of CO_2 emissions on the supply chain and marketing strategy of automobile manufacturers (Atasu et al., 2008).

Then, we have considered a comprehensive literature review for different approaches regarding the asset (fleet) replacement problem which is the core idea of this thesis. Although the machine or vehicle replacement literature is rich in models dealing with budget constraints (Chand et al., 2000; Karabakal et al., 1994), stochastic demands (Hartman, 2001), and heterogeneous types of vehicles (Hartman, 2004), these models have not been considered risk management perspective.

Finally, we have provided a general introduction and survey of risk management and risk measuring and its applications especially in operations. We have also considered a general discussion of risk management issues in operations and supply chains. Moreover, we have defined the concepts of risk measuring, coherent risk measure, VaR, and CVaR.

In summary, from the comprehensive literature review in this Chapter, it is evident that there is a gap that this article aims to address: to explain sustainable fleet replacement from an uncertainty perspective using risk management methodologies.

Next, Chapter 3 addresses the issue of fleet replacement problem taking into account the some of the challenges that we have mentioned. Specifically, we consider the uncertainties due to fuel and CO_2 pries, fuel consumption, and mileage driven by vehicles in a two-stage decision making model and CVaR.

CHAPTER 3

A TWO-STAGE STOCHASTIC PROGRAMMING MODEL FOR SUSTAINABLE FLEET REPLACEMENT

Au chapitre 3, nous effectuons un examen de la littérature consacrée à la valeur à risque conditionnelle (CVaR). C'est une mesure du risque cohérente que nous utilisons pour la gestion du risque dans le contexte du remplacement du parc. Puis nous proposons un modèle de programmation stochastique à deux étapes pour une politique de remplacement du parc. Il s'agit d'une politique statique où les décisions sont prises à la première étape et nous étudions un type de véhicule de différentes marques. Nous donnons également certains résultats analytiques pour une comparaison de la CVaR des véhicules à carburant fossile et des véhicules électriques, en prenant en compte la volatilité du prix du CO2 et du carburant, la consommation de carburant et la distance parcourue. Enfin, nous validons ces résultats analytiques par une étude de cas réel et nous formulons une conclusion à ce chapitre.

In this Chapter, we provide a two-stage stochastic programming model for the replacement policy. By two stages we mean we adopt a static policy in which the optimal decisions for leasing a vehicle, VaR, and CVaR are made at first year of planning horizon. So, it is different from conventional two stage stochastic programming and we do not make decisions in two stages. We also present some analytical results for comparing the CVaR of different technologies of fossil fuel vehicles and EVs taking into account the uncertainties, which are: CO_2 and fuel price volatility, fuel consumption, and mileage driven by a vehicle. Finally, we validate the analytical results using a real case study and we conclude the Chapter.

The Chapter is organized as follows: Section 3.1 introduces the modeling of fleet replacement and develops a customized two-stage stochastic mixed-integer linear programming model (MILP) for minimizing risk and expected cost. Section 3.2 presents the analytical results on the comparison of the CVaR of different types of vehicles. Section 3.3 provides a case study for validating the analytical results, and Section 3.4 describes the results of a real case study. Lastly, Section 3.5 presents the summary of the Chapter.

3.1 A Two-Stage Model for Fleet Replacement Policy

This section presents a stochastic model for vehicle leasing in a given planning horizon. The aim is to obtain the optimal policy that minimizes the cost and the risk simultaneously. Because equation (3.1) uses a two-stage model, the average fuel prices (f_u) are calculated. In equation (3.1), f_{ut} denotes the forecasted fuel prices during the planning horizon, and T is the length of the horizon in years. Moreover, equation (3.2), to obtain the average electricity charge costs of batteries of EVs, e_{bu}, (for different brands with respect to a benchmark brand for 100 miles) uses the average electricity prices from equation (3.1). In equation (3.2), p_b is the ratio of battery capacity to the benchmark brand. The notations are summarized in Table 3.1(a).

$$f_u = \sum_{t=1}^{T} f_{ut} / T \qquad \text{For all } u \qquad (3.1)$$

$$e_{bu} = f_u p_b \qquad \text{For all } u \text{ and } b \qquad (3.2)$$

Next, equation (3.3) describes how to compute the running cost for each brand of fossil fuel technology, per 100 km, under different scenarios. In addition, the running cost for electric vehicles (EVs), per 100 km, is calculated using equation (3.4). In equation (3.3), o_{bv} denotes fuel consumption for fossil fuel vehicles per 100 km. The cost of CO_2 emissions for different technologies, in (3.3) and (3.4), is taken into account by including parameter c_p^s, which shows carbon prices in different states of the world. Other parameters in (3.3) and (3.4) are described in Table 3.1(b).

Table 3.1(a): The indices and decision variables, for technologies, brands, and different scenarios for carbon and fuel prices, mileage driven, and fuel consumption

i=1, 2 index for fossil fuel and electrical technologies, respectively

b=1, 2, B index for brands for b_1, b_2, and benchmark brand (b_B), respectively

t=1, 2, .., T index for number of periods in the year

s=1, 2, S index for the state of carbon prices for low, medium, and high, respectively

u=1, 2,.., U index for the forecasted fuel price scenarios from of 2012 to 2016

m=1, 2,.., M index for scenarios from the distribution of monthly mileage driven by a car

v=1,….,V index for scenarios from the distribution of fuel consumption by a car

x_{ib}: The car with technology i and brand b that has been leased

z_{ibusmv}: Auxiliary stochastic variables for the loss function

α_{ib}^{β} : Value at risk at confidence level β for a car with technology i and brand b

ϕ_{ib}^{β}: Conditional value at risk at confidence level β for a car with technology i and brand b

Table 3.1(b): Parameters of the model

W: Conversion coefficient of mileage to km

ω: Parameter for trade-off of risk and cost in the objective function

β: Confidence level for calculating CVaR and VaR

B_b: Annual lease cost for batteries of EVs with brand b

p_b: Ratio of the capacity of the battery of EVs with brand b to the benchmark brand (22 kw)

c_p^s : The expected CO_2 prices for each state s

c^e: The CO_2 emissions (gr) per km for electrical technology

c^g: The CO_2 emissions (gr) per liter for fossil fuel technology

l_{ib}: The monthly lease cost for each technology i with brand b.

D_m: The monthly mileage driven by a car for each scenario m

o_{bv}: Fuel consumption per 100 km for brand b and each scenario v

f_{ut}: Forecasted fuel prices for each scenario u in year t

f_u: Average fuel prices for each scenario u during the planning horizon

e_{bu}: Average charge cost of EV batteries for 100 miles with brand b for each scenario u

r_{ibusv}: The running cost per 100 km for technology i with brand b for each scenario u, each state of carbon price s, and each scenario for fuel consumption v

y_{ibusmv}: The total running cost per technology i with brand b for scenario u, each state of carbon price s, each scenario for monthly mileage driven by cars m, and each scenario for fuel consumption v

μ_{ib}: The total fixed cost per technology i with brand b

$$r_{ibusv} = o_{bv}(f_u + c_p^s c^g / 10^6) \qquad \text{For all } b, u, s, v, \text{ and } i=1 \qquad (3.3)$$

$$r_{ibusv} = e_{bu} / W + 100(c^e / 10^6) c_p^s \qquad \text{For all } b, u, s, v, \text{ and } i=2 \qquad (3.4)$$

Consequently, equation (3.5), based on equations (3.3) and (3.4), describe how to compute the total running cost over the planning horizon. In equation (3.5), D_m represents the monthly mileage driven by a vehicle.

$$y_{ibusmv} = 48(r_{ibusv} / 100)WD_m \qquad \text{For all } i, b, u, s, v, \text{ and } m \tag{3.5}$$

Lastly, equations (3.6) and (3.7) are used to calculate the total investment (fixed) cost for fossil fuel technologies (3.6) and for EVs (3.7). The monthly leasing cost, which is shown by l_{ib}, is used to obtain total fixed costs in the planning horizon. Moreover, for EVs, there is an extra investment cost, which is the annual lease cost for batteries, B_b.

$$\mu_{ib} = 48 l_{ib} \qquad \text{For all } b \text{ and } i=1 \tag{3.6}$$

$$\mu_{ib} = 48 l_{ib} + 4B_b \qquad \text{For all } b \text{ and } i=2 \tag{3.7}$$

The objective is to minimize the weighted average of CVaR and the total expected cost. The decision variable is x_{ib}, which denotes a vehicle with technology i and brand b. By combining the formulas and parameters presented in the previous sections, the stochastic mixed integer programming problem is represented by equations (3.8)-(3.13).

$$\underset{x\in\{0,1\},z\in R,\alpha_{ib}^{\beta}\in R}{Min} = \omega\ E(\text{cost})\ + (1\text{-}\omega)\phi_{ib}^{\beta} \tag{3.8}$$

$$E(\text{cost}) = \sum_{i=1}^{2}\sum_{b=1}^{B}\mu_{ib}x_{ib} + (\sum_{i=1}^{2}\sum_{b=1}^{B}\sum_{u=1}^{U}\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{v=1}^{V}y_{ibusmv}x_{ib})\ /USMV \tag{3.9}$$

$$\phi_{ib}^{\beta} = \alpha_{ib}^{\beta} + 1/(USMV(1-\beta))\sum_{u=1}^{U}\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{v=1}^{V}z_{ibusmv} \qquad \text{For all } i \text{ and } b \tag{3.10}$$

$$z_{ibusmv} \geq (\mu_{ib} + y_{ibusmv})x_{ib} - \alpha_{ib}^{\beta} \qquad \text{For all } i, b, u, s, m, \text{ and } v \tag{3.11}$$

$$\sum_{i=1}^{2}\sum_{b=1}^{B}x_{ib} = 1 \tag{3.12}$$

$$x_{ib} \in \{0,1\} \tag{3.13}$$

Because the objective of this stochastic program is to minimize the cost and risk simultaneously, equation (3.8) is used to minimize the weighted average of the total expected cost, E (cost), and CVaR. That is, by changing the value of parameter ω to different combinations of the total expected cost, the risks over the planning horizon are minimized, depending on whether the focus is more on cost or on risk. Equation (3.9) is used to calculate the expected total cost, which includes the fixed cost and running cost. The running cost is calculated based on the realization of all of the stochastic processes for each brand and technology. Moreover, equations (3.10)-(3.11) are used to compute the value of CVaR at confidence level β (Rockafellar and Uryasev, 2000). In inequality (3.11), the first term on the right-hand side denotes the loss function (Rockafellar and Uryasev, 2000), and it is related to the total expected cost for different scenarios. Lastly, equations (3.12)-(3.13) are the constraints on the decision variable. Solving (3.8)-(3.13), depending on the value of ω, yields the optimal vector x^*, corresponding VaR*, optimal CVaR*, and total expected cost.

3.2 Analytical Results on the Comparison of the CVaR of Different Technologies

This section presents the analytical results comparing the CVaR of different technologies. Let y_{ibus} denote the stochastic total running cost for technology i with brand b, taking into account u scenarios for fuel prices and s states for carbon prices.

Proposition 3.1: *The $\phi^\beta(y_{2bus})$ for EVs is less than the $\phi^\beta(y_{1bus})$ for fossil fuel vehicles if and only if this condition holds:* $\phi^\beta(f_u) - \phi^\beta(e_{bu})/(Wo_b) \geq c_p{}^s(c^g/10^6 - c^e/(o_b 10^4))$.

Proof: As $\phi^\beta(.)$ is a coherent risk measure (Artzner et al., 1999):

$$\phi^{\beta}(hX) = h\phi^{\beta}(X) \tag{3.14}$$

$$\phi^{\beta}(X+h) = \phi^{\beta}(X) - h \tag{3.15}$$

$$\phi^{\beta}(X+Y) \leq \phi^{\beta}(X) + \phi^{\beta}(Y) \tag{3.16}$$

In equations (3.14)-(3.16), h is an arbitrary constant and X and Y denote stochastic variables. Moreover, these equations are referred to as the Positive Homogeneity, Translation Invariance, and Subadditivity properties of coherent risk measures, respectively (Artzner et al., 1997). The comparison of the $\phi^{\beta}(.)$ for EVs with fossil fuel vehicles is based on the stochastic total running cost. Based on equations (3.3)-(3.5) and (3.14)-(3.15), equation (3.17) can be derived.

$$\phi^{\beta}(y_{1bus}) = 0.48WDo_b\phi^{\beta}(f_u) - 0.48o_bWDc_p{}^sc^g/10^6 \qquad \text{For all } b, u, s \tag{3.17}$$

Specifically, equation (3.17) is for the case considering fossil fuel prices as stochastic processes. In equation (3.17), D denotes the expected monthly mileage driven by a car and is a constant parameter (Table 3.4). Moreover, for the case of EVs in equation (3.18) using properties (3.14)-(3.15), the value of $\phi^{\beta}(.)$ is calculated.

$$\phi^{\beta}(y_{2bus}) = 0.48D\phi^{\beta}(e_{bu}) - 48WDc_p{}^sc^e/10^6 \qquad \text{For all } b, u, s \tag{3.18}$$

By subtracting (3.18) from (3.17), which is denoted by k, the relationship in (3.19) is obtained.

$$\begin{aligned}&0.48WDo_b\phi^{\beta}(f_u) - 0.48D\phi^{\beta}(e_{bu}) - 0.48o_bWDc_p{}^sc^g/10^6 + \\ &48WDc_p{}^sc^e/10^6 \geq 0\end{aligned} \tag{3.19}$$

By dividing both sides of inequality (3.19) by $0.48WD$, it follows that $\phi^{\beta}(f_u) - \phi^{\beta}(e_{bu})/(o_b W) \geq c_p{}^s(c^g/10^6 - c^e/(o_b 10^4))$.

Then, setting $k' = c^g/10^6 - c^e/(o_b 10^4)$ yields (3.20).

$$\phi^{\beta}(f_u) - \phi^{\beta}(e_{bu})/(o_b W) \geq c_p{}^s k'. \qquad \blacksquare \tag{3.20}$$

The right-hand side of inequality (3.20) is a positive number. The left-hand side is a stochastic variable because its value depends on the various realizations of fuel prices in different scenarios. To illustrate, the Figure 3.1 represents inequality (3.20).

As can be seen in Figure 3.1, there are four areas in which the relationship between different values of ϕ^{β}(.) for different stochastic processes is presented. Specifically, the feasible solution area that is shown by (Δ_1) and (Δ_2) is the area in which condition (3.20) holds. However, in the parts that are shown by (Δ_3) and (Δ_4), there is no feasible solution for inequality (3.20). The interesting point about area (Δ_2) is that the value ϕ^{β}(.) for fossil fuel prices is smaller than that for electricity prices, but it is higher for the stochastic total running cost. The reason is the different slopes and positive number ($k' c^s{}_p$), which are shown in Figure 3.1. As a result, in these areas, the value of k would be positive, and $\phi^{\beta}(y_{2bus})$ for EVs is less than the $\phi^{\beta}(y_{1bus})$ for fossil fuel vehicles.

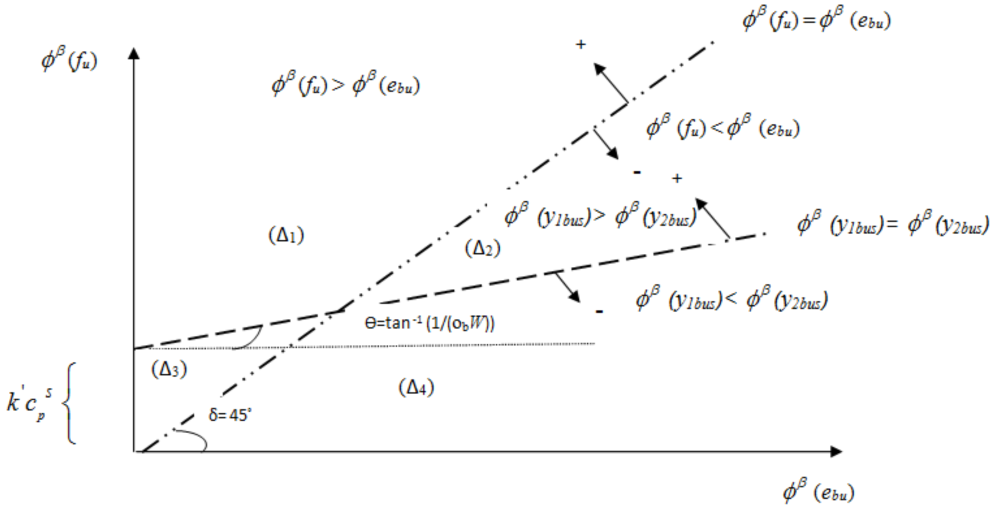

Figure 3.1: The feasible solution for Proposition 3.1, which is shown by (Δ_1) and (Δ_2)

Let y_{ibsm} denote the stochastic total running cost for technology i with brand b, taking into account m scenarios for mileage driven and s states for carbon prices, and let f denote the average fuel price during the planning horizon.

Proposition 3.2: *The* $\phi^\beta(y_{2bsm})$ *for EVs is less than the* $\phi^\beta(y_{1bsm})$ *for fossil fuel vehicles if and only if this condition holds*:

$$e_b - Wo_b f < c_p^s Wo_b(c^g/10^6 - c^e/(o_b 10^4)).$$

Proof: The stochastic total running cost in which stochastic parameters exist is used to compare $\phi^\beta(.)$ for the EVs and for the fossil fuels. Therefore, based on equations (3.3)-(3.5) and property (3.14), it follows that, for all b, m, s:

$$\begin{aligned}\phi^{\beta}(y_{1bsm}) &= \phi^{\beta}(0.48 r_{1bs} W D_m) = 0.48 W r_{1bs} \phi^{\beta}(D_m) \\ &= 0.48 W \phi^{\beta}(D_m)(o_b f + o_b c_p{}^s c^g / 10^6)\end{aligned} \quad . \tag{3.21}$$

Specifically, equation (3.21) holds for fossil fuel vehicles, considering mileage driven as the stochastic process. Moreover, for the case of EVs, the value of $\phi^{\beta}(.)$ is obtained using property (3.14) and represented by equation (3.22), for all *b, m, s*:

$$\begin{aligned}&\phi^{\beta}(y_{2bsm}) = \phi^{\beta}(0.48 r_{2bs} W D_m) = 0.48 W r_{2bs} \phi^{\beta}(D_m) = \\ &0.48 W \phi^{\beta}(D_m)(e_b / W + 100 c_p{}^s c^e / 10^6)\end{aligned} \tag{3.22}$$

Therefore, by comparing (3.22) and (3.21), we derived the inequality (3.23).

$$\phi^{\beta}(y_{2bsm}) < \phi^{\beta}(y_{1bsm}) \Leftrightarrow e_b - W o_b f < c_p{}^s W o_b k' \ . \quad \blacksquare \tag{3.23}$$

In inequality (3.23), both sides are real numbers depending on different values of parameters and the values of *f* and e_b, which are the expected values of fuel prices and electricity prices, respectively. For illustration, inequality (3.23) is represented in Figure 3.2. The feasible solution areas are represented by (Δ_1) and (Δ_2). Indeed, in these areas, the condition in Proposition (3.2) is true, and ϕ^{β} (y_{2bsm}) for EVs is less than the ϕ^{β} (y_{1bsm}) for fossil fuel vehicles.

Let y_{ibsv} denote the stochastic total running cost for technology *i* with brand *b*, taking into account *v* scenarios for fuel consumption and *s* states for carbon prices.

Proposition 3.3: *The* $\phi^{\beta}(y_{2bsv})$ *for EVs is less than the* $\phi^{\beta}(y_{1bsv})$ *for fossil fuel vehicles if and only if this condition holds:* $\phi^{\beta}(o_{bv}) > \dfrac{(e_b / W + c_p{}^s c^e / 10^4)}{(f + c_p{}^s c^g / 10^6)}$.

Proof: The stochastic total running cost is used to compare the $\phi^{\beta}(.)$ of EVs with the $\phi^{\beta}(.)$ of EVs of fossil fuel vehicles. Based on equations (3.3)-(3.5) and properties (3.14) and (3.16), inequality (3.24) is obtained.

$$\phi^{\beta}(y_{1bsv}) \leq 0.48WD\phi^{\beta}(o_{bv})(f + c_p{}^s c^g / 10^6) \quad \text{For all } b, v, s \tag{3.24}$$

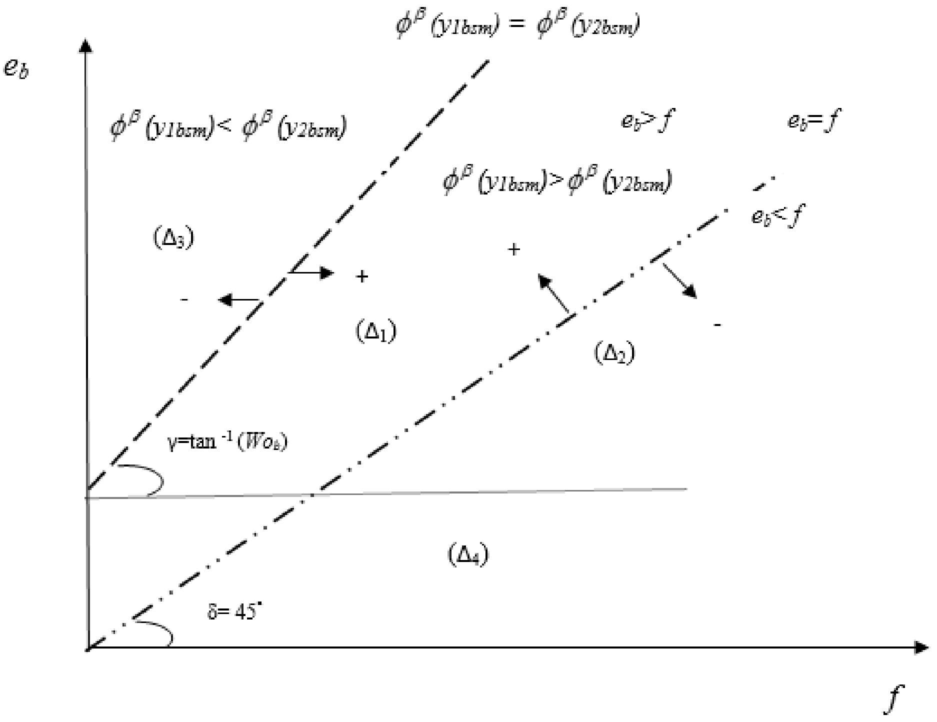

Figure 3.2: The feasible solution for Proposition 3.2, which is shown by (Δ_1) and (Δ_2).

Inequality (3.24) is obtained based on the subadditivity property (3.16), and it holds for fossil fuel vehicles when considering fuel consumption as the stochastic process. In addition, for the case of EVs, the value of $\phi^{\beta}(.)$, represented by equation (3.25), is obtained from property (3.14). So by comparing (3.24) and (3.25), after some basic algebra, inequality (3.26) is derived. The right-hand side of inequality (3.26) is a positive number, between one and two, depending on the different values of parameters and the values of f and e_b, which are expected values of fuel prices and electricity prices, respectively. However, the left-hand side is a stochastic variable depending on the realization of different scenarios for the fuel consumption of fossil fuel technologies per 100 km. Inequality (3.26) is illustrated in Figure 3.3.

$$\phi^{\beta}(y_{2bsv}) = 0.48WD(e_b / W + 100{c_p}^s c^e / 10^6) \qquad \text{For all } b, v, s \tag{3.25}$$

$$\phi^{\beta}(o_{bv}) > \frac{(e_b / W + {c_p}^s c^e / 10^4)}{(f + {c_p}^s c^g / 10^6)} \quad . \ \blacksquare \tag{3.26}$$

Because f and e_b are correlated, $\phi^{\beta}(o_{bv})$ is represented as a function of f. As represented in Figure 3.3, the graph is a decreasing homographic function with a horizontal asymptote k'' equals to $e_b / (W f)$. The intuition behind this pattern is that by increasing the expected fuel prices, the $\phi^{\beta}(o_{bv})$ will decrease due to lower fuel consumption. Specifically, the feasible solution area, which is represented by (Δ_1) and (Δ_2), is the area in which condition (3.26) holds. Therefore, it follows that the $\phi^{\beta}(y_{2bsv})$ for EVs is less than the $\phi^{\beta}(y_{1bsv})$ for fossil fuel vehicles.

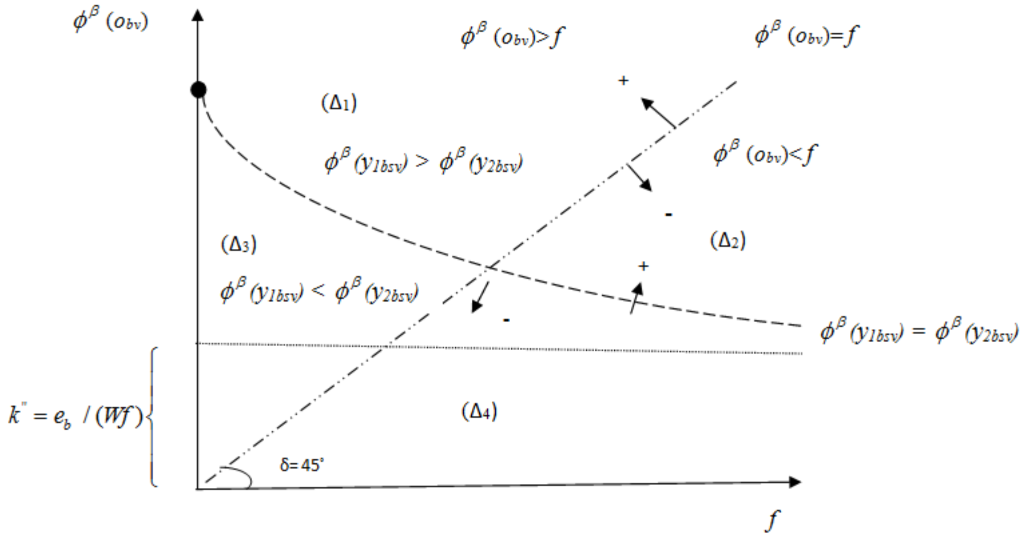

Figure 3.3: The feasible solution for Proposition 3.3, which is represented by (Δ_1) and (Δ_2)

The application of the analytical results in this Chapter, as will be represented in the case study, gives us the general intuition that risk of EVs in comparison with other technologies is lower for each stochastic process.

3.3 A Case Study on Sustainable Fleet Management

An important issue when developing a model is to determine whether it is an accurate representation of the system studied, i.e., if it is valid (Landry et al., 1983; Law and Kelton, 1991; Landry and Oral, 1993).

The term "accurate representation" is used to mean the extent to which the model fits the real system either in terms of structure and mechanism or in terms of output, depending on the context of the problem. The validity of the model was ensured in using the following steps. The case presented in this section was based on: 1) real data from the fleet analyzed, including mileage and consumption per vehicle; 2) real data on the leasing costs for different types of vehicles and brands; 3) forecasts for the fuel prices for the planning horizon considered, based on real data; and 4) a model for CO_2 prices estimated from real data.

Moreover, the validity of the model is also tested by comparing the decisions recommended by the model with the current fleet used by the company. This is reported in Section 3.4 for the case in which only expected values were used: in this case, as is currently the case, the optimal decision is to lease diesel vehicles only.

The goal is to obtain the optimal policy for vehicle replacement, using leasing, by considering a planning horizon of four years (2012 to the beginning of 2016). Three fuel technologies (Petrol, Diesel, and Electricity) and three brands (b_1, b_2, and b_B,) are considered; b_B is the benchmark. Even though they are based on real vehicles, for the purpose of anonymity, the brands are denoted as such. The typical consumption of the benchmark brand is 7.6 liters/100 km for diesel and 9.3 liters/100 km for petrol. A current petrol price of approximately £1.37/liter and diesel price of £1.41 /liter are assumed. The cost for leasing the battery of the electric vehicle, for the benchmark brand, is £950 per year in the UK, and the cost to charge is £2.5 per charge (for 100 miles autonomy). (In this study, electricity and electricity charge prices are used interchangeably. However, indeed, the price of electricity is the price of each charge for the 22 kWh battery of the benchmark brand).

Therefore, to obtain the electricity charge for other brands, the ratio of the power of the battery with respect to the battery of the benchmark brand (Table 3.2) can be used. The emissions in the UK are estimated to be approximately 81 g/km (benchmark's estimate), and for petrol and diesel, the emissions are estimated to be approximately 2310 and 2680 g/liter, respectively. Moreover, for carbon prices, because there is no clear historical trend, three states of prices (low prices, £5, medium prices, £10, and high prices, £20) are used. The other parameters for other brands, including the benchmark brand are presented in Tables 3.2 and 3.3.

Table 3.2: The parameters for electric vehicles with different brands

Brand	Cost of renting the battery per year(£)	The ratio of the battery of each brand to the benchmark brand (22
b_1	1100	1.6
b_2	1050	1.3
b_B	950	1

Table 3.3: The leasing costs and fuels consumption for cars with different brands

Technology/Brand	Monthly lease cost (£)	Fuel consumption per 100 km (liter)
Petrol-b_1	230	9.8
Diesel-b_1	240	7.9
Electric-b_1	450	No fuel consumption
Petrol- b_2	210	9.1
Diesel- b_2	220	6.9
Electric-b_2	400	No fuel consumption
Petrol-b_B	220	9.3
Diesel-b_B	230	7.6
Electric-b_B	380	No fuel consumption

Given all of the assumptions about the electric version of the benchmark brand, it follows that they are less competitive in comparison with the diesel and petrol version of it when the annual expected mileage driven is less than 19526 miles/year, with the last assumptions and monthly leasing costs of £220, £230, and £380 for petrol, diesel, and EVs, respectively. As represented in Figure 3.4, the total costs, including the running and investment cost for EVs, with last assumptions about the benchmark brand's parameters, are less than other technologies when the total annual mileage is above the intersection of the diesel and electric lines. Moreover, petrol cars are more competitive in comparison with the other two technologies when the average annual mileage driven is less than 2843 miles/year. For diesel cars, it is economical to use them when the average annual mileage is between 2843 and 19526 miles. These thresholds depend on fuel and carbon prices and monthly leasing costs.

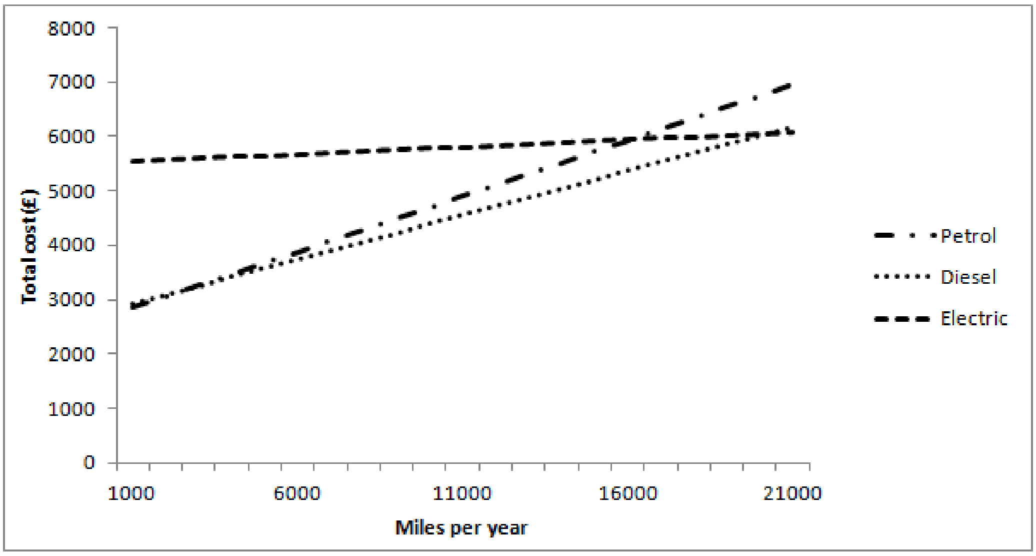

Figure 3.4: The total cost (running plus investment cost) versus the average mileage driven in one year for different technologies.

3.3.1 Vector Auto Regression for Forecasting Fuel Prices

The historical data used for fuel prices is based on a time series from Jan. 2000 to Dec. 2011 in UK (Available online at: automotive association, 2014; government, 2014). Because the fuel prices are correlated (Table 3.4), the method used in this Chapter to forecast fuel prices is Vector Auto Regression (e.g, Widiarta et al., 2007). The Vector Auto Regression is used in forecasting systems of interrelated time series for analyzing the dynamic impact of random disturbances on the system of variables.

Table 3.4: Correlation matrix for fuel prices from Jan. 2000 to Dec. 2011

	Petrol	Diesel	Electric
petrol	1.00	0.99	0.85
Diesel	0.99	1.00	0.88
Electric	0.85	0.88	1.00

The Vector Auto Regression approach treats every endogenous variable as a function of the lagged values of all of the endogenous variables in the system. The mathematical representation of Vector Auto Regression is the following:

$$Y_t = A_1\, Y_{t-1} + \ldots + Ap\, Y_{t-p} + BX_t + e_t$$

where Y_t is a vector of endogenous variables, X_t is a vector of exogenous variables, $A_1, A_2, \ldots, A_p$ and B are matrices of coefficients to be estimated, and e_t is a vector of white noises that may be contemporaneously correlated but are uncorrelated with their own lagged values and uncorrelated with all of the right-hand side variables.

Because fuel prices are not stationary, first-order differentiation is used to convert them to a stationary process, Akaike (1977), Fuller (1976), Lütkepohl (1991), Schwarz (1978).

As seen from Table 3.5, based on the Augmented Dickey-Fuller test, Fuller (1976), the differentiated fuel prices with one order differentiation are stationary because the Null Hypothesis, which suggests that the differentiated fuel price has a unit root, is rejected.

Table 3.5: Results for differentiated fuel prices with one order differentiation

Null Hypothesis: D(Petrol) has a unit root		**t-statistic**	**Prob.**
Augmented Dickey-Fuller test statistic		-7.86	0.00
Test critical values:	1% level	-3.47	
	5% level	-2.88	
	10% level	-2.57	
Null Hypothesis: D(Diesel) has a unit root		t-statistic	Prob.
Augmented Dickey-Fuller test statistic		-7.22	0.00
Test critical values:	1% level	-3.47	
	5% level	-2.88	
	10% level	-2.57	
Null Hypothesis: D(Electricity) has a unit root		t-statistic	Prob.
Augmented Dickey-Fuller test statistic		-8.01	0.00
Test critical values:	1% level	-3.47	
	5% level	-2.88	
	10% level	-2.57	

The next step is to compute various criteria to select the lag order of VAR. Table 3.6 displays various information criteria for all lags up to the specified maximum. The criterion that has the lowest value between different Lags should be selected. Based on Table 3.6, because the Schwarz Information Criterion (SC) and the Akaike information criterion (AIC), which have similar definitions, Schwarz (1978) and Akaike (1977), show different lag orders, the third criterion, which is the Hannan-Quinn information criterion (HQ), is also considered. The HQ criterion (Hannan and Barry, 1979) has the lowest value for the lag 1 between different lags; as a result, VAR with lag order equals one is used.

Table 3.6: Different values for criteria for choosing the order of Lag

Vector Auto Regression Lag Selection Criteria			
Lag	AIC	SC	HQ
0	-14.56	-14.49	-14.53
1	-14.98	**-14.71**	**-14.87**
2	-14.86	-14.4	-14.67
3	-14.86	-14.2	-14.59
4	-14.93	-14.08	-14.59
5	**-15**	-13.95	-14.57
6	-14.93	-13.68	-14.42
7	-14.88	-13.44	-14.3
8	-14.86	-13.22	-14.19

In the next section, the AR root's graph (Lütkepohl, 1991) is obtained. The estimated VAR is stable (stationary) if all roots have a modulus less than one and lie inside the unit circle. If the VAR is not stable, certain results are not valid. There will be kp roots, where k is the number of endogenous variables and p is the largest lag. Therefore, based on the fact that there are three endogenous variables, which are petrol, diesel, and electricity, and the largest lag order is one (Table 3.6), there should be three roots. As can be seen in Figure 3.5, all of the roots are inside the unit circle, and the estimated VAR is stable (Lütkepohl, 1991). Lastly, the coefficients for simultaneous equations of VAR are shown in Table 3.7.

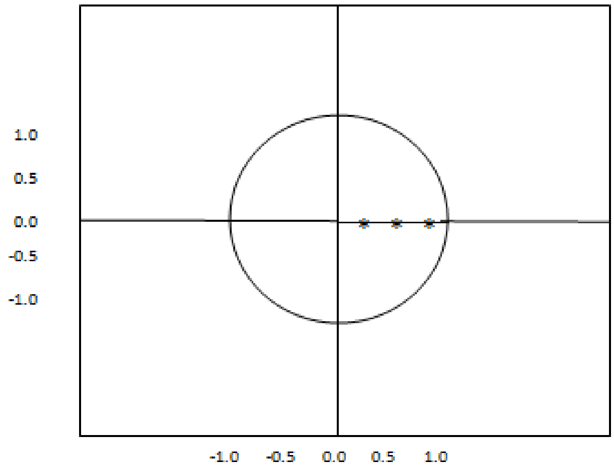

Figure 3.5: Unit circle for testing the stability of estimated VAR

Table 3.7: The coefficients for solving the VAR model for fuel prices for a sample of data from Jan. 2000 to Dec. 2011

Vector Auto Regression Estimates Standard error in ( ) and t-statistics in [ ]			
	D(Petrol)	D(Diesel)	D(Electricity)
D(Petrol(-1))	0.112	-0.077	0.044
	(0.173)	(0.160)	(0.239)
	[0.646]	[-0.484]	[0.187]
D(Diesel(-1))	0.261	0.506	-0.201
	(0.184)	(0.170)	(0.254)
	[1.41]	[2.96]	[-0.79]
D(Electric(-1))	-0.211	-0.123	0.354
	(0.059)	(0.054)	(0.080)
	[-3.57]	[-2.25]	[4.34]
C	0.004	0.003	0.006
	(0.002)	(0.001)	(0.002)
	[2.09]	[1.81]	[2.11]
R-squared	0.22	0.22	0.143
Log Likelihood	327.46	338.27	282.94
Akaike AIC	-4.654	-4.809	-4.01
Schwarz SC	-4.56	-4.72	-3.92
Mean dependent	0.004	0.004	0.008
S.D. dependent	0.02	0.02	0.03

Let p_t, d_t, and e_t denote petrol, diesel, and electricity prices at time t, then:

$$p_t = 0.004 + p_{t-1} - 0.211(e_{t-1} - e_{t-2}) + \varepsilon_{tp} \tag{3.27}$$

$$d_t = 0.003 + 1.506 d_{t-1} - 0.506 d_{t-2} - 0.12(e_{t-1} - e_{t-2}) + \varepsilon_{td} \quad (3.28)$$

$$e_t = 0.006 + 1.354 e_{t-1} - 0.354 e_{t-2} + \varepsilon_{te} \quad (3.29)$$

$$\varepsilon_{tp} = N(0, \sigma_p),\ \varepsilon_{td} = N(0, \sigma_d),\ \varepsilon_{te} = N(0, \sigma_e)$$

In equations (3.27)-(3.29), each fuel price is described as a function of the significant lagged values of two other fuel prices and its own white noise (ε_t) that are uncorrelated with their own lagged values and all of the right-hand side variables. Moreover, these equations show that by considering two additional fuel prices as endogenous variables in the main equation for forecasting each of them, the correlation between the fuel prices is taken into account (Table 3.4). Equations (3.27)-(3.29) are used in forecasting fuel prices over the planning horizon.

3.3.2 Modeling Uncertainty about the Driven Mileage

As presented in Figure 3.6, in the dataset of 2789 vehicles, the monthly mileage driven follows a lognormal distribution with the mode at approximately 500 miles per month. In this sample, 9.2% of the vehicles had zero mileage during the period analyzed.

As seen in Table 3.8 the mean mileage is approximately 834 miles per month, and the median is 750 miles per month. This result implies that 50% of the vehicles are used less than 750 miles per month. For considering the mileage driven by a car, the scenarios are generated using a lognormal distribution with its parameters estimated to fit the data (Evans et al., 2000).

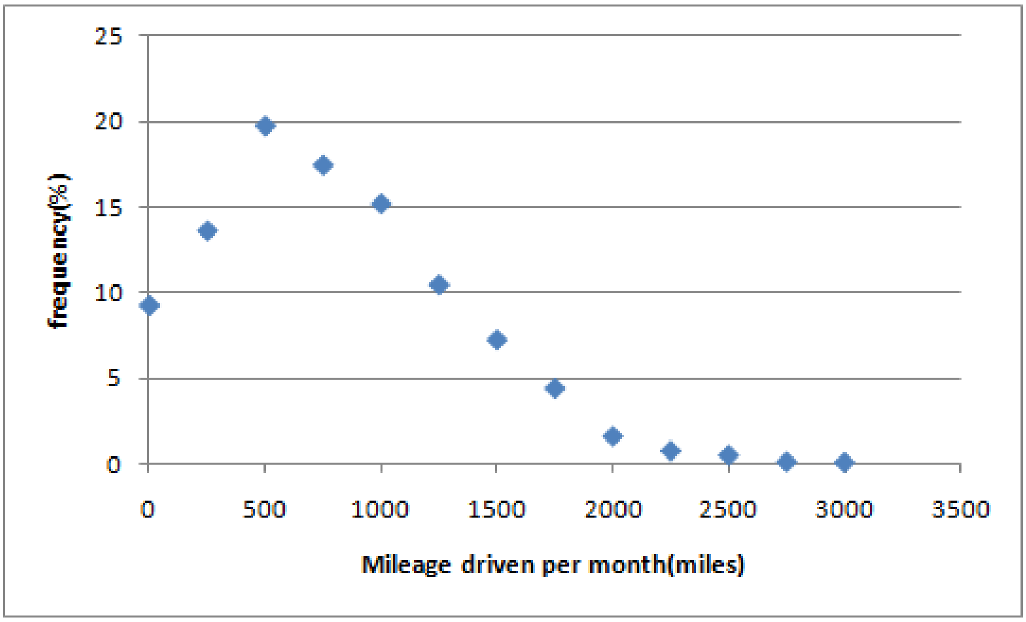

Figure 3.6: Actual distribution of monthly mileage driven by cars based on the data.

Table 3.8: Descriptive statistics for the monthly mileage driven by cars

Mileage driven per month	Miles
Mean	834.27
Mode	500.00
Median	750.00
Std. Deviation	510.17

3.3.3 Modeling Uncertainty about Fuel Consumption

Another stochastic parameter that is considered in the analysis is fuel consumption per 100 km, both by diesel and petrol cars. Because fuel consumption depends on the different conditions under which the vehicles are used (e.g., motorways vs. urban areas) and the skill of the driver, it is essential to consider it as a stochastic process. As shown in Figure 3.7, the fuel consumption per 100 km, based on the real data used in the present study, follows a lognormal distribution. In this case, our data include 2789 diesel vehicles with a consumption mode at approximately 5 liters/100 km; in the period under analysis, 13.6% of the vehicles had an average consumption of approximately 4 liters per 100 km.

As seen in Table 3.9, the mean of fuel consumption is approximately 6.9 liters per 100 km, and the median is 6.22 liters. This result implies that 50% of the vehicles have less than 6.22 liters consumption per 100 km. Therefore, for considering fuel consumption by a car, the scenarios can be generated using a lognormal distribution with its parameters estimated to fit the data (Evans et al., 2000).

We should also mention that the total mileage that should be driven by vehicles in each month (D_m) regardless of technology is the same for petrol, diesel, and electric vehicles. But, the fuel consumption per 100km for petrol and diesel are different based on Table 3.3. So, based on competitive advantage of each technology in terms of saving in running cost which is a function of fuel prices, carbon prices, amount of emissions, and fuel consumption (equations 3.3, 3.4, and 3.5), and fixed cost which is the function of monthly lease cost and battery prices for EVs (equations 3.6, and 3.7), the optimal technology is selected by the output of the model.

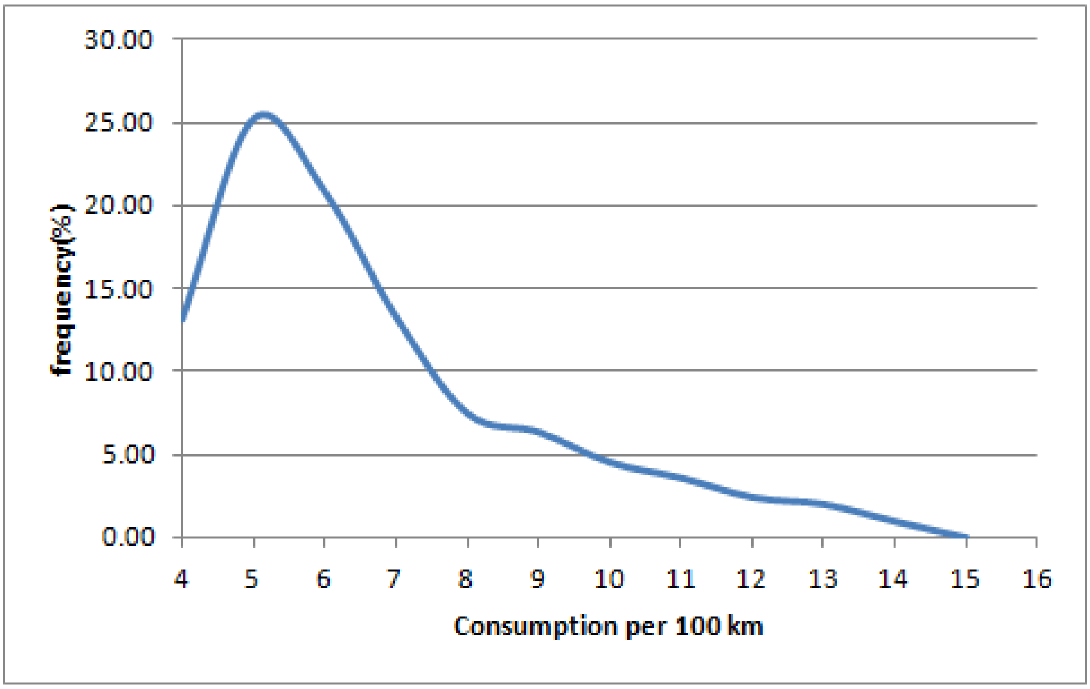

Figure 3.7: Actual distribution of fuel consumption by cars per 100 km based on the data

Table 3.9: Descriptive statistics for the fuel consumption per 100 km by a car

Consumption per 100 km	liter
Mean	6.9
Mode	5.00
Median	6.22
Std. Deviation	2.3

3.4 Case Study Results

This section presents the results of the case study, first taking into account each stochastic process separately and then analyzing their joint effect on the cost and risk associated with each different type of vehicle. The generation of the fuel price scenarios for each technology is made with equation (3.1). As a two-stage problem is considered and decisions are made at the first stage, the average of fuel prices is obtained for each scenario during the planning horizon. Moreover, for the other two stochastic processes, which are mileage driven and fuel consumption, the scenarios are generated based on a fitted distribution, which is a lognormal with the parameters matched with the data, as explained in Sections 3.3.2 and 3.3.3.

In each set of simulations, when considering each stochastic process separately, a total of 12000 scenarios (4000 scenarios for each of them and 3 states for carbon prices) are used. Moreover, when considering all of the stochastic processes simultaneously, due to the higher complexity of the model, 24000 scenarios (20 scenarios for fuel prices, 20 scenarios for mileage driven, 20 scenarios fuel consumption, and 3 states for carbon prices) are used.

The number of scenarios is obtained based on trial and error for convergence of the model. (It has been determined that if the number of scenarios increases, the results will be not be changed. It has also been verified that if they decrease, there will be an inconsistency problem). In addition, the confidence level β equals 0.9. The planning horizon is assumed to be four years from the beginning of 2012 to the beginning of 2016, which is the standard leasing duration of the vehicles.

Regarding the interpretation of the CVaR in the following tables, it should be noted that there is no bad or good CVaR. Indeed, the CVaR itself represents the expected loss faced by the company with a given probability. A way to better interpret the CVaR is to consider the gap between expected cost and CVaR as a measure of risk. The lower this gap, the lower the risk. Moreover, the value of CVaR also depends on the confidence level (β). The closer the value of β is to 1, the higher the values of

CVaR: in this case, the fleet manger is more conservative. We also have compared the results with risk neutral formulation. So, using CVaR improves the model for risk measuring over the traditional models that only take into account the minimization of the expected cost.

First, the impact of fuel prices on the choice of the vehicle to be leased is analyzed. If the expected values for mileage driven presented in Table 3.8 and fuel consumption depicted in Table 3.9 are taken into account, then the optimal choice of vehicle, as a function of the weights of the expected cost and of the CVaR, is summarized in Table 3.10.

In Table 3.10, when ω changes from 0 to 0.7, i.e., the focus is on minimizing risk rather than minimizing expected cost, the optimal policy is to choose an electrical vehicle from the benchmark brand (b_B). In contrast, by increasing the weight of the expected cost, i.e., ω ranges from 0.7 to 1, the best option is to lease the diesel vehicle, and b_2 is the chosen brand due to its better capital and running costs. Moreover, by choosing a diesel vehicle, there is a reduction in expected cost of approximately £ 5.91 K (25.49%) and an increase in the associated CVaR by £ 16.06 (59%) K. That is, by choosing a diesel vehicle, there is an increase in the risk of approximately £ 0.4 per mile for the expected mileage over the planning horizon due to the volatility in fuel prices.

Furthermore, the expected value of the monthly mileage driven by cars, which is 834 miles per month (Table 3.8), is taken into account. As mentioned in Section 3.3, if this value decreases to approximately 250 miles per month, then rather than a diesel vehicle, a petrol vehicle will be the optimal choice for minimizing the cost. In contrast, if this value increases to approximately 1700 miles per month, then the electric vehicle will be chosen rather than the diesel vehicle (Figure 3.4). However, for minimizing risk, the electrical vehicle is always the optimal choice regardless of the expected value of the monthly mileage driven in the model (Proposition 3.1).

Table 3.10: Results for considering the fuel prices, in 000£, as a stochastic process in the model

ω	**0**	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**	**1**
weighted-	27.22	26.82	26.02	25.20	24.40	19.88	17.27
expected-	23.18	23.18	23.18	23.18	23.18	17.27	17.27
CVaR(K£)	27.23	27.23	27.23	27.23	27.23	43.29	51.00
VaR(K£)	27.06	27.06	27.06	27.06	27.06	43.29	51.00
Technology	Electric	Electric	Electric	Electric	Electric	Diesel	Diesel

Next, the impact of mileage uncertainty on the optimal choice of vehicle is considered. Assuming that the expected values for fuel prices are in Table 3.11, the associated risks (due to fuel price uncertainty) and costs for a vehicle are computed for the planning horizon. The results are depicted in Table 3.12. When the value of ω is within the range 0 to 0.7 the electric vehicle is the optimal choice. However, for values of ω above 0.7 the diesel vehicle is chosen instead. Moreover, by choosing the diesel vehicle as the optimal choice we have a reduction in total cost which is about £ 5.84 K (25.15%) and increasing the associated CVaR by £ 42.13 K (127.4%). Therefore, leasing an electric car for four years can mitigate the risk due to uncertainty in the mileage driven. A formal proof is provided in Proposition 3.2.

Table 3.11: Expected fuel prices (£) from 2012 to the beginning of 2016

	2012	2013	2014	2015	Average (four years)
Petrol(£)	1.37	1.40	1.42	1.45	1.41
Diesel(£)	1.41	1.45	1.50	1.55	1.48
Electric(£)	2.54	2.65	2.76	2.87	2.70

Table 3.12: Results for considering the mileage driven by car, in 000£, as the stochastic process in the model for four years

ω	**0**	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**	**1**
weighted-cost(K£)	33.06	32.08	30.10	28.13	26.16	23.15	17.36
expected-cost (K£)	23.20	23.20	23.20	23.20	23.20	17.36	17.36
CVaR (K£)	33.07	33.07	33.07	33.07	33.07	75.20	167.00
VaR (K£)	30.16	30.16	30.16	30.16	30.16	58.38	167.00
technology brand	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Diesel b_2	Diesel b_2

Next, the impact of fuel consumption in the choice of vehicle is considered. The benchmark fuel consumption for each technology and its brand (Table 3.3) and the fitted standard deviation for each brand are used to generate the scenarios. The expected values for fuel prices (Table 3.11) and mileage driven (Table 3.8) are assumed. The results are summarized in Table 3.13. As seen by changing the values of ω from 0 to 0.7, it is optimal to choose the electric vehicle for minimizing the risk and cost simultaneously. However, if the value of ω increases more than 0.7 up to 1, then the diesel vehicle is the optimal choice for minimizing cost. Moreover, by choosing the diesel vehicle, there is a reduction in total cost of approximately £ 6.08 K (26.24%) and an increase in the associated CVaR of approximately £ 29.08 K (108.55%). As a result, leasing an electric vehicle significantly decreases the risk due to volatility in fossil fuel consumption. A formal proof for this issue is provided in Proposition 3.3.

Table 3.13: Results for considering fuel consumption by petrol and diesel cars, in 000£, as the stochastic process in the model for four years

ω	0	0.1	0.3	0.5	0.7	0.9	1
weighted-cost(K£)	26.79	26.43	25.71	24.99	24.27	20.97	17.10
expected-cost (K£)	23.18	23.18	23.18	23.18	23.18	17.10	17.10
CVaR (K£)	26.79	26.79	26.79	26.79	26.79	55.87	96.00
VaR (K£)	26.79	26.79	26.79	26.79	26.79	48.38	96.00
technology brand	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Diesel b_2	Diesel b_2

Another important issue is the ranking of risk drivers in the model. As seen by comparing Tables 3.10, 3.12, and 3.13, the diesel vehicle (brand b_2) and the electric vehicle (brand b_B) are the optimal choices based on different values of ω. Indeed, if you are more risk averse, you choose electric technology, and if you are more risk neutral, you choose diesel technology with the corresponding brands as the optimal choices. However, the petrol vehicle and brand b_1 and are not competitive with the aforementioned technologies and brands in terms of risk or cost minimization. This is why only the risk drivers of diesel and electric vehicles with associated optimal brands are considered in Figures 3.8 and 3.9, respectively.

As seen from Figure 3.8, the most important risk driver when a diesel vehicle is used with brand b_2 is mileage driven, which has the highest value of CVaR, followed by fuel consumption and, finally, by fuel prices. This surprising result is very specific to the data, and it is justified by the large volatility in the distribution of fuel consumption presented in Figure 3.7.

Furthermore, in Figure 3.9, the value of CVaR for different risk drivers is represented when EVs of the benchmark brand are used. In this case, the fuel price ranked as the second most important risk factor for EVs in terms of the value of

CVaR. In addition, when Figures 3.8 and 3.9 are compared in terms of the value of CVaR, as mentioned before in Propositions 3.1, 3.2, and 3.3, the value of CVaR for diesel vehicles is higher than for EVs for each corresponding stochastic process.

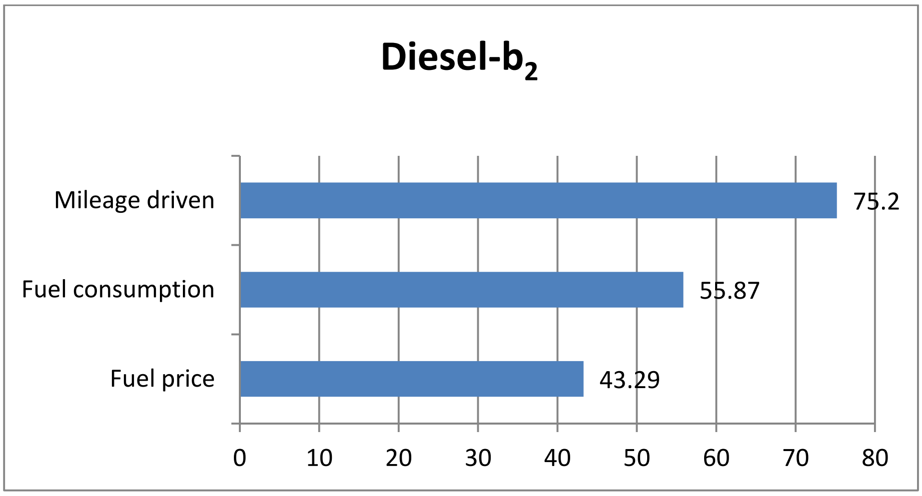

Figure 3.8: Comparing risk drivers in terms of value of CVaR, in 000£, from 2012 to 2016 for diesel technology with brand b_2.

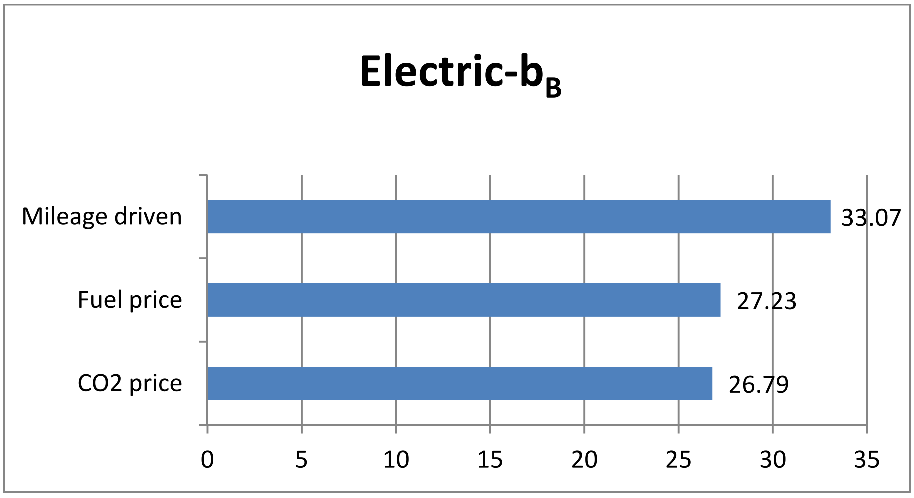

Figure 3.9: Comparing risk drivers in terms of the value of CVaR, in 000£, for four years from 2012 to 2016 for benchmark brand (b_B).

Now, the complete stochastic model when there is uncertainty due to fuel prices, mileage driven, and fuel consumption is considered. The results for the full model are presented in Table 3.14. As seen by changing values of ω from 0 to 0.7, the optimal decision is to lease an electric vehicle. However, if the value of ω increases more than 0.7 up to 1, then the diesel vehicle is the best option. Moreover, by choosing the diesel vehicle as the optimal choice, there is a reduction in total cost, which is approximately £ 6.26 K (27.12%), and an increase in the associated CVaR by £ 34.52 K (116.58%). Therefore, as a general conclusion, it seems that leasing an electric vehicle can significantly mitigate risk exposure at an additional expected cost.

One important conclusion, when comparing the CVaR by considering all stochastic processes in the model with the case when only one stochastic process is considered separately is that the CVaR when all of the stochastic processes are considered is less than sum of the CVaRs for the stochastic processes separately. This result is supported by the subadditivity property of coherent measures, as presented in equation (3.30), Artzner et al. (1997). Therefore, by taking into account inequality (3.30) and (3.31), it can be concluded that the analytical results are supported by the computational results in Tables 3.10, 3.12, 3.13, and 3.14.

$$\phi(X+Y+Z) \le \phi(X)+\phi(Y)+\phi(Z) \tag{3.30}$$

$$\phi_{ib}^{\beta}(f_{iu}, D_m, o_{bv}) \le \phi_{ib}^{\beta}(f_{iu}) + \phi_{ib}^{\beta}(D_m) + \phi_{ib}^{\beta}(o_{bv}) \tag{3.31}$$

Lastly, the total cost per mile for each mileage scenario (per month), for the b_2 Diesel vehicle and b_B EV are considered. As seen from Figure 3.10, the total cost per mile has a decreasing trend as the average mileage driven increases per month in each scenario. Indeed, for high-mileage vehicles, both trends converge to £ 0.34 per mile. However, for normal expected mileage, which is 834 miles per month (Table 3.8), there is a difference of approximately £ 0.15 per month between the two choices. Therefore, it follows that if the high-mileage case is considered (Section 3.3, Figure 3.4) and other stochastic processes are included in the decision support model (i.e., fuel prices and fuel consumption), the EV is the optimal choice.

Table 3.14: Results for considering fuel prices, mileage driven, and fuel consumption, in 000£, as stochastic processes in the model for four years

ω	**0**	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**	**1**
weighted-cost(K£)	29.61	28.96	27.65	26.35	25.04	21.55	16.82
expected-cost (K£)	23.08	23.08	23.08	23.08	23.08	16.82	16.82
CVaR (K£)	29.61	29.61	29.61	29.61	29.61	64.13	98.00
VaR (K£)	28.96	28.96	28.96	28.96	28.96	55.00	98.00
technology brand	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Diesel b_2	Diesel b_2

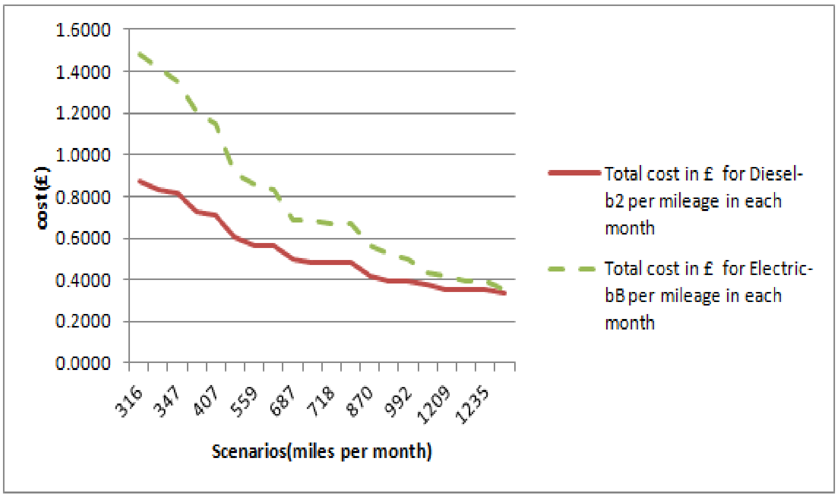

Figure 3.10: Total cost in £ per mileage for diesel technology with brand b_2 and electric technology with the benchmark brand for each scenario of mileage driven in each month

3.5 Summary

Fleet management is an important tool for reducing CO_2 emissions and fuel costs and improving transportation sustainability. This Chapter proposes a stochastic mixed integer linear programming model that incorporates risk concerns (CVaR) to analyze the choice of technology by a firm that aims to replace some of its vehicles. The firm minimizes expected cost and risk simultaneously, taking into account the uncertainties that exist in the real situation: carbon prices, fuel prices, mileage driven, and fuel consumption.

Specifically, the analytical results show that for each stochastic process of fuel prices, mileage driven, and fuel consumption, the value of CVaR for EVs is less than for fossil fuel vehicles under certain conditions. For example, for the case involving fuel prices treated as a stochastic process, leasing a diesel vehicle rather than an electric vehicle increases the value of CVaR by 59%. This value for mileage driven and fuel consumption is 127.4% and 108.6%, respectively. In addition, the results show that if each stochastic process is considered separately, the most important risk driver for a diesel vehicle is the mileage driven, followed by fuel consumption, and lastly, fuel prices. For the case of EVs, the first important risk factor is mileage, followed by fuel prices and then CO_2 prices.

Furthermore, when all of the stochastic processes are considered together, leasing a diesel vehicle rather than an electric vehicle for four years (2012 to 2016) decreases the total expected cost by approximately £ 6.26 K (27.13%) and increases the associated risk by £ 34.52 K (116.6%) due to uncertainty in the carbon prices, fuel prices, mileage driven, and fuel consumption. Moreover, by considering all stochastic processes together, it can be seen that the risk of the whole model is less than the summation of risk for each stochastic process.

Lastly, by comparing the total cost per mile for each mileage scenario (per month) and including other uncertainty factors in the decision support model, it can be concluded that for high-mileage vehicles, the EV is the optimal choice.

In the next Chapter, we consider the fleet replacement problem in a dynamic setting. Moreover, we present the concept of time consistency for a dynamic risk measure and we discuss it for CVaR. Then, by using clustering method and using it in real data, we test whether clustering can decrease CVaR and expected cost.

CHAPTER 4

A MULTI-STAGE STOCHASTIC PROGRAMMING MODEL FOR MANAGING A SUSTAINABLE FLEET PORTFOLIO SYSTEM

Au chapitre 4, nous prolongeons le travail du chapitre 3 dans une configuration à étapes multiples. Dans ce contexte, les décisions sont mises à jour à chaque période au cours de laquelle l'interaction entre les différents types de véhicule aux différentes capacités a été examinée à l'aide de l'analyse de concentration. Dans ce chapitre, comme contribution méthodologique, nous proposons une nouvelle formulation récursive de la CVaR, qui est cohérente dans le temps, dans un cadre dynamique. En outre, grâce à l'analyse de concentration, nous tenons compte de l'effet de portefeuille de l'utilisation de différentes technologies, sur le système du parc, au niveau de la CVaR et du coût escompté. Enfin, nous exposons les résultats analytiques afin de mesurer l'effet de portefeuille et la cohérence dans le temps d'une nouvelle formulation de la CVaR et nous appliquons le modèle dans une étude de cas réel.

In this Chapter, we extend the model in Chapter three to a multi stage setting. In this context the decisions are updated at every period (Figure 4.1). By multiple stages we mean a dynamic policy in which we can find the optimal policies at each node of scenario tree for the decisions variables which are the optimal number of new leased vehicles, value at risk, and CVaR based on realization of stochastic processes. The problem is formulated by a Mixed Inter Programming (MIP) model. In terms of the difference of the model in this Chapter with existing fleet management models which makes it unique, because of the following reasons:

(1) We have compared the results of risk neutral formulation (ω=1), and risk averse formulation (ω=0 and ω=0.5) which is the advantage of using this model over existing models in the literature. (2) In the literature none of fleet management models have taken into account the risk and cost minimization, simultaneously in the objective function. (3) It has taken into account the time consistency of CVaR in a dynamic setting with a new recursive formulation. (4) The computational results also in this chapter show the importance of using clustering in the model with different technologies and capacities and their impact on the risk and expected cost in the fleet management system.

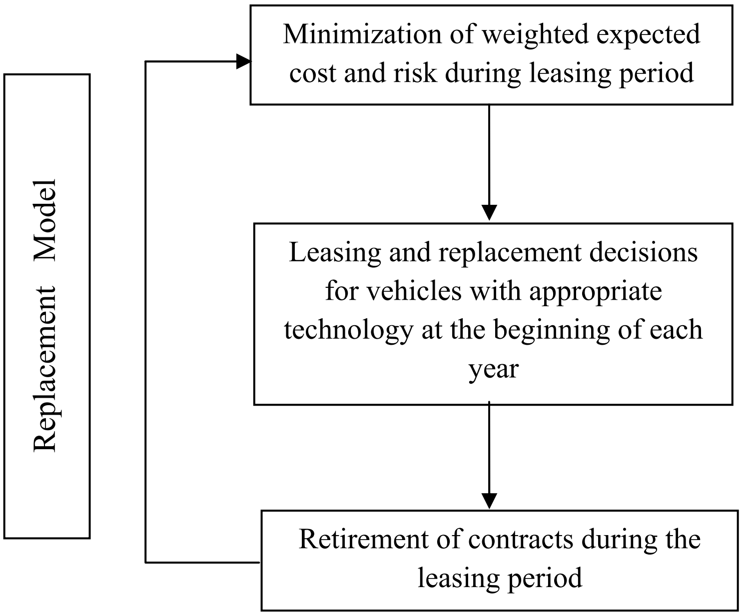

Figure 4.1: The replacement model for fleet managers.

The remainder of this Chapter proceeds as follows. In Section 4.1 we develop a multi-stage stochastic programming model for minimising the weighted average of the expected cost and risk, considering the existing constraints and uncertainties in the market. In Sections 4.2 and 4.3, we derive the analytical results for the introduced time consistent version of CVaR. In Sections 4.4 and 4.5, we present a real case study and, finally, we present the main conclusions in Section 4.6.

4.1 A General Model for the Management of Fleet System

In this section, we introduce a multi-stage stochastic programming model to obtain the optimal number of vehicles to be leased, taking into account the constraints that exist to minimise a cost function that considers expected cost and CVaR during the planning horizon. The notation used is summarized in Tables 4.1(a) and 4.1(b). In Figure 4.2, at each node, we have a vector of stochastic processes, namely, fuel prices, CO_2 prices, mileage driven, and fuel consumption for fossil fuel technologies, per 100 km. We consider five technologies: fossil fuels (petrol, diesel), hybrids (petrol, diesel), and EVs.

Table 4.1(a): Indices and decision variables

$i \in I$={fossil fuels, hybrids, and electric}
$a \in A$= {1, 2,.., A} index for age of the vehicles
$n \in N$= {1,2,…,N }index for nodes in scenario tree
$t \in T$= {1, 2, ..,T}index for time periods in year over planning horizon
$s_t \in S$= {1, 2,.., S_t } index for number of branches (states) at each stage
$k \in K$={1, 2,.., K} index for different clusters of vehicles
$\Psi_{n,m}$: Tree structure for parent nodes n and child nodes m
$\Omega_{t,n}$: Structure for the association of each node n and each stage of t
x_{nia}: Total number of vehicles with technology i and age a currently leased at node n
y_{ni}: Number of new vehicles with technology i that company leases at node n
α_n^{β} : Value at risk at confidence level of β at node n
ϕ_n^{β} : Conditional value at risk at confidence level of β at node n
z_n: Auxiliary stochastic variables for loss function at node n

In equation (4.1), we calculate the running cost for fossil fuel and hybrid vehicles per 100 km, r_{ni}, at each node; in this equation o_n denotes the fuel consumption of fossil fuels and hybrids per 100 km at each node. The running cost for EVs per 100 km, r_{ni}, is calculated using equation (4.2). We also take into account fuel prices for each technology (f_{ni}) and CO_2 emissions at each node (c_n^p) in (4.1) and (4.2). Furthermore, c^g denotes the CO_2 emissions (g/litre) for fossil fuels and hybrids, and c^e shows the CO_2 emissions for EVs (g/km). Finally, W represents the conversion coefficient from miles to kilomteres.

Table 4.1(b): Parameters of the model.

W: Conversion coefficient of mileage to km

ω: Parameter for trade-off of risk and cost in the objective function

β: Confidence level for calculating CVaR and VaR

G_k: Number of vehicles in cluster k

L_n: Loss function at node n

h_{na}: Initial condition of the fleet system with fossil fuel technology at node n, age a

f_{ni}: Fuel price for technology i at node n

o_n: Fuel consumption at node n

D_n: Monthly mileage driven at node n

Q_n: Expected cost function at node n

r_{ni}: Running cost per 100 km for technology i at node n

c_n^p: CO_2 prices at each node n

c^e: CO_2 emissions, g per km, for electrical technology

c_i^g: CO_2 emissions, g per litre, for fossil fuel and hybrid technology

l_i: Monthly lease cost for each technology i

M_e: Monthly lease cost for EV batteries

λ_{ni}: Total annual running cost for technology i at node n

μ_i: Total annual fixed cost for technology i

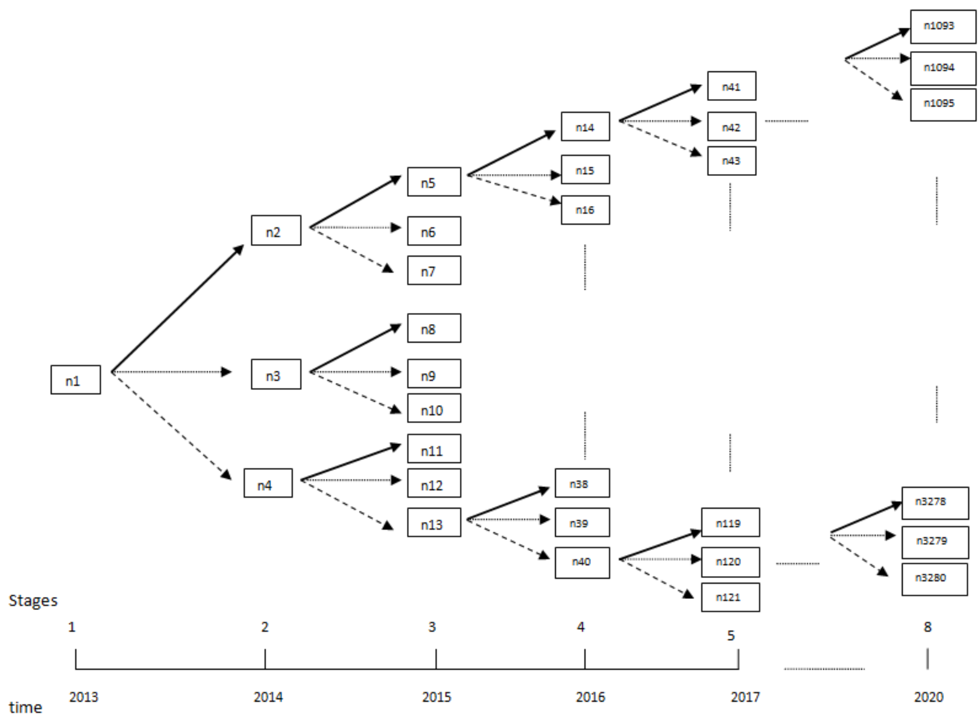

Figure 4.2: The node-based tree model and corresponding stages (3 branches at each node).

$$r_{ni} = o_n(f_{ni} + c_n^p c_i^g / 10^6) \qquad \forall n \in N \text{ , } i\text{=fossil fuels, hybrids} \qquad (4.1)$$

$$r_{ni} = \frac{f_{ni}}{W} + 100(\mathrm{c}^e / 10^6) c_n^p \qquad \forall n \in N \text{ , } i\text{=electric} \qquad (4.2)$$

As a result, based on equations (4.1) and (4.2), we calculate the total running cost at each node, λ_{ni}, using equation (4.3), in which D_n represents the monthly mileage driven at node n.

$$\lambda_{ni} = 12Wr_{ni} / 100D_n \qquad \forall n \in N, i \tag{4.3}$$

The total investment cost is represented by (4.4) for fossil fuels and hybrid technologies and by (4.5) for EVs. Because we take into account the leasing contracts for providing different vehicle types in the fleet system, we use the monthly lease cost, represented by l_i, to obtain the fixed cost at each node. Moreover, for EVs we have an extra investment cost, which is the monthly lease cost for batteries, as presented in equation (4.5) by M_e.

$$\mu_i = 12l_i \qquad i = \text{fossil fuels, hybrid} \tag{4.4}$$

$$\mu_i = 12(l_i + M_e) \qquad i = \text{electric} \tag{4.5}$$

Our objective is to minimise the weighted average of CVaR and cost at the root node. The decision variable is y_{ni}, which denotes the number of vehicles with technology i that are replaced, at each node, at the beginning of each year due to the retirement of contracts for leasing of the vehicles. Each firm aims to solve the mixed integer multi-stage stochastic programming model in equations (4.6)-(4.17).

$$\underset{x_{nia}, y_{ni}, \alpha_n^\beta}{Min} \quad \omega Q_1 + (1-\omega)\phi_1^\beta \tag{4.6}$$

s.t.

$$x_{1ia} = h_{na} \quad \forall a \in A, i = \text{fossil fuel} \tag{4.7}$$

$$y_{ni} = x_{ni1} \quad \forall n \in N, i \in I \tag{4.8}$$

$$y_{ni} = 0 \qquad \forall n \in \Omega_{t,n} \text{ if } t \geq T\text{-}3 \tag{4.9}$$

$$x_{mia} = y_{mi} + x_{ni(a-1)} \quad \forall a \in A, \forall (n,m) \in \Psi_{n,nn} \tag{4.10}$$

$$\sum_i \sum_a x_{nia} \geq \sum_a h_{na} \quad \forall n \in N \tag{4.11}$$

$$L_n = \sum_i \sum_a (\lambda_{ni} + \mu_i) x_{nia} / 10^6 \qquad \forall n \in \mathrm{N} \tag{4.12}$$

$$Q_n = L_n + \frac{1}{S_t} \sum_{\Psi(n,m)} (Q_m) \qquad \forall (n,m) \in \Psi_{n,m} \tag{4.13}$$

$$z_m \geq L_m - \alpha_n^{\beta} \qquad \forall (n,m) \in \Psi_{n.m} \tag{4.14}$$

$$v_m \geq Q_m - \alpha_n^{\beta} \qquad \forall (n,m) \in \Psi_{n,m} \tag{4.15}$$

$$\phi_n^\beta = \alpha_n^\beta + \frac{1}{S_t(1-\beta)} \sum_{\psi(n,m)} z_m + \frac{1}{S_t} \sum_{\psi(n,m)} \phi_m \quad (4.16)\ (a)$$

$$\phi_n^\beta = \alpha_n^\beta + \frac{1}{S_t(1-\beta)} \sum_{\psi(n,m)} v_m \quad (4.16)\ (b)$$

$$\alpha_n^\beta = 0 \qquad \forall \mathrm{n} \in \Omega_{t,n} \quad (4.17)$$

$$x_{nia}, y_{ni} \in N\{0\}, \text{and } \ \alpha_n^\beta, z_m, \ and \ v_m \in R^+ \quad (4.18)$$

The objective function (4.6) minimizes the weighted average of expected cost, Q_1, and CVaR, ϕ_1^β, at the root node. Obviously, if ω equals 1, only the expected cost is minimized and if ω is equal to zero, only CVaR is minimized. Equation (4.7) shows that the initial condition of the fleet system, h_{ia}, which comprises fossil fuel vehicles of different ages, should be equal to total number of the vehicles, x_{1ia}, at the root node. In equation (4.8) we determine the number of new leased vehicles at each node, y_{ni}, required to replace the segment of new vehicles of the total vehicles in the fleet system, x_{ni1}, due to retirement of the older vehicles at the corresponding node. In addition, equation (4.9) shows that the planning horizon for decision variable y_{ni} is four years, after which there will be no new leased vehicles in the fleet system. Equation (4.10) shows that the total number of vehicles at each child node, x_{mia}, is equal to the number of new leased vehicles, y_{mi}, plus the number of vehicles, x_{nia}, in the predecessor node. Moreover, equation (4.11) represents that the total number of vehicles for all technologies and ages, $\sum_i \sum_a x_{nia}$, at each node should be greater than or equal to the number of vehicles needed, $\sum_a h_{na}$, at the corresponding node.

Equation (4.12) shows the loss function (total cost), L_n, at each node, which is composed of the running cost, λ_{ni}, and fixed cost, μ_i, at the corresponding node. In addition, equation (4.13) represents the recursive formula for calculating the expected cost function at each node, Q_n, which is equal to the loss function, L_n, at the corresponding node plus the average of cost functions, $\frac{1}{S_t}\sum_{\Psi(n,m)}(Q_m)$, in the successor nodes. To take into account the time consistency issue of CVaR (Shapiro, 2011), we use equations in (4.16). In (4.16) we have defined two different risk measures. The first risk measure, i.e., (4.16) (a), take into account the effect of CVaR at the child nodes and we name it Recursive Expected CVaR (RECVaR). The second one, i.e., (4.16) (b) is adapted from Shapiro (2011). In addition, equation (4.17) shows that VaR, α_n^{β}, at the final stage should be zero because there is no uncertainty at this stage and all the values of the stochastic processes are realized. Finally, (4.18) is the constraint for the integer, x_{nia} and y_{ni}, and non-negative, α_n^{β}, z_n, and v_m decision variables.

4.2 Time Consistency of Dynamic Risk Measure

In the recent literature, time consistency is represented to be one important requirement to get appropriate optimal decisions, for multistage stochastic programming models. Papers on time consistency are categorized in two different approaches: the first one concentrates on risk measures and the second one on optimal policies (Rudloff et al., 2014).

The first approach says that, in a dynamic setting, if some random payoff A is always riskier than a payoff B conditioned to a given time t+1, then A should also be riskier than B conditioned to t. It is shown that this property leads to so called time consistent dynamic risk measure suggested by several authors, e.g., Detlefsen et al., (2009). Other definitions, such as acceptance and rejection consistency, are also developed in the literature, e.g., Cheridito et al. (2006); Kovacevic and Pug (2009).

The second approach, defined by Shapiro (2009), is on time consistency of optimal policies in multistage stochastic programming models. Consider a T-stage scenario tree representing the evolution of the corresponding data process (Figure 4.2). This scenario tree represents a finite number of possibilities of what can happen in the future. At stage (time) $t = 1$, we have one root node denoted by n_1. At stage $t \geq 2$, we have S_t branches at each node n (in Figure 4.2, we have assumed S_t to be equal to 3). Each branch is connected to the predecessor node by an arc. By $\Omega_{t,n}$ we denote the set of all nodes at stage t=1, 2,..., T. Moreover, $\xi_{ni} = (c_n^p, f_{ni}, D_n, o_n)$ denotes the random process for the realisation of the stochastic parameters of CO_2 prices (c_n^p), fuel prices for each technology (f_{ni}), mileage driven (D_n), and fuel consumption (o_n) at each node n and stage t. In addition, we denote $\Psi_{n,m}$ as the set of nodes and children nodes. The children nodes (m) of node n at stage t are nodes that can happen at the next stage t+1. A scenario representing a particular realisation of the data process is a sequence of nodes and children nodes, such that $(n,m) \in \Psi_{n,m}$.

Thus far we have not said anything about the optimality of our decisions. At every node (state) $n \in \Omega_{t,n}$ of the fleet system at time t, we have information about the past, i.e., we know the history of the process from the root node to the current state. A basic concept of the multistage stochastic programming model is the requirement of *nonanticipativity* (Shapiro, 2009). That is, our decisions should be a function of the history of the data process available at the time t, when decisions are made. We also have an idea of which scenarios could and could not happen in the future. Thus, it is natural to consider the conceptual requirement that an optimal decision at state n should not depend on states that do not follow n, i.e., cannot happen in the future. That is, the optimality of our decision at state n should only involve future children nodes (m) of state n. This principle is called *time consistency* (Shapiro, 2009). This time consistency requirement is closely related to, although not the same as, the Bellman's principle used to derive dynamic programming equations. The standard risk neutral formulation of multi-stage programming problems satisfies this principle. On the other hand, some risk-averse stochastic programming problems do not satisfy this requirement (Shapiro, 2009). Other alternatives have been proposed by Boda and Filar (2006) and Cuoco et al. (2008), however none of them used the recursive set up of time consistent dynamic risk measures.

Now we are ready to consider the time consistency of the risk measure in our fleet model presented in Section 4.1. The static value of ϕ^{β}, Rockafellar and Uryasev (2000), is obtained by the following system for the confidence level β. In addition, γ_s represents the discrete scenarios sampled from the distribution of the stochastic processes in the model, S is the number of scenarios, $v=\left[(1-\beta)S\right]^{-1}$, x is the vector of decision variables, z_s are positive variables, and L denotes the loss function. Solving the following LP model gives the optimal value of $\phi^{\beta*}$, the decision variable x^*, and $\alpha^{\beta*}$.

$$\min_{x,\alpha^{\beta}} \phi^{\beta} = \min_{x,\alpha^{\beta}} \ \alpha^{\beta} + v\sum_{s=1}^{S} z_s$$

s.t.

$z_s \geq L(x,\gamma_s)-\alpha^{\beta} \quad z_s \geq 0,\ s=1,2,\ldots,S$

It has been shown by Boda and Filar (2006) and Shapiro (2009) that, in general, in a dynamic setting the above model for calculating ϕ^{β} is not time consistent. For this reason, we present a dynamic risk measure of ϕ^{β}, denoted by ϕ_n^{β}, equation (4.16) (a), which is the value of ϕ^{β} at each node $n \in \Omega_{t,n}$, and we show it is time consistent. Our approach is different from Shapiro (2011), equation (4.16) (b), where the concept of cost-to-go functions was used to satisfy the time consistency principle, as we provide a recursive formulation of the CVaR for a scenario tree, explicitly computing the CVaR of the parent node as a function of the CVaRs and expected conditional expectations of the extreme cost of the respective children nodes. It also differs from Boda and Filar (2006), in which the target-percentile approach was applied for to follow the time consistency principle.

For a better grasp of constraints (4.16) (a, b), we have represented a numerical example as follows. In addition, in the context of the overall optimization problem we can define the risk measures in (4.16) (a, b) as:

$$\phi_n^{\beta} = \underset{\alpha_n^{\beta}}{Min}\left(\alpha_n^{\beta} + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)} z_m + \frac{1}{S_t}\sum_{\psi(n,m)} \phi_m\right) \tag{4.19}$$

$$\phi_n^{\beta} = \underset{\alpha_n^{\beta}}{Min}\left(\alpha_n^{\beta} + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)} v_m\right) \tag{4.20}$$

We assume a three stage scenario tree that has four branches at each stage. We consider level of β equals 50%. The losses at the final nodes are represented in Figure 4.3. The losses at stage 2 are L_2=0, L_3=10, L_4=20, and L_5=80. Moreover, all the values of CVaRs at the final nodes are zero. We use equations (4.14)-(4.15), and (4.19)-(4.20) for calculation of CVaR at different nodes.

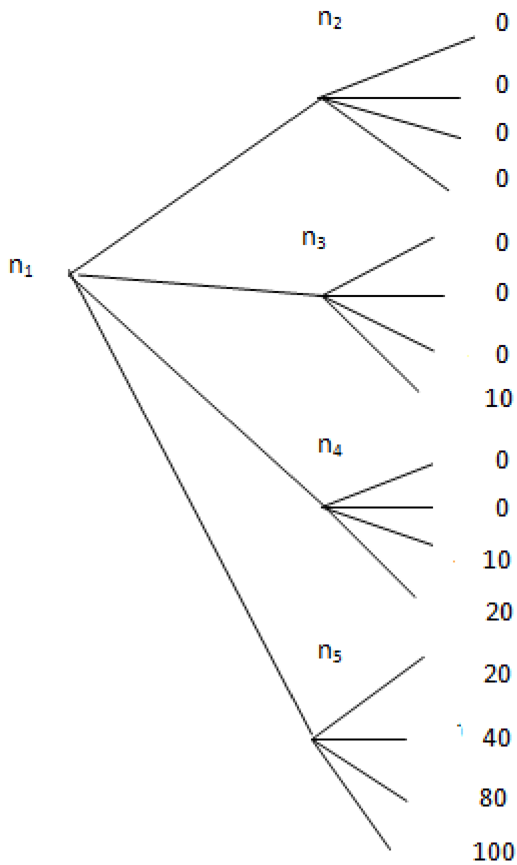

Figure 4.3: A three stage scenario tree with S_t= 4 at each stage.

By using equation (4.19), first we obtain the RECVaRs at stage 2, which are the nodes 2, 3, 4, and 5, i.e, $\phi_2^\beta, \phi_3^\beta, \phi_4^\beta$, and ϕ_5^β.

$$\phi_2^\beta = \underset{\alpha_2^\beta}{Min}\ [\alpha_2^\beta + \frac{1}{4(1-0.5)}[(0-\alpha_2^\beta)^+ + (0-\alpha_2^\beta)^+ + (0-\alpha_2^\beta)^+ + (0-\alpha_2^\beta)^+]]$$
$$= \underset{\alpha_2^\beta}{Min}\ [\alpha_2^\beta + 2(0-\alpha_2^\beta)^+]$$

Then by minimizing ϕ_2^β we have: ϕ_2^β=0, and α_2^β=0. Next we calculate ϕ_3^β.

$$\phi_3^\beta = \underset{\alpha_3^\beta}{Min}\ [\alpha_3^\beta + \frac{1}{4(1-0.5)}[(0-\alpha_3^\beta)^+ + (0-\alpha_3^\beta)^+ + (0-\alpha_3^\beta)^+ + (10-\alpha_3^\beta)^+]]$$

$$= \underset{\alpha_3^\beta}{Min}\ [\alpha_3^\beta + 1.5(0-\alpha_3^\beta)^+ + \frac{1}{2}(10-\alpha_3^\beta)^+]$$

If we minimize ϕ_3^β we have: ϕ_3^β=5, and α_3^β=0. Next we calculate ϕ_4^β.

$$\phi_4^\beta = \underset{\alpha_4^\beta}{Min}\ [\alpha_4^\beta + \frac{1}{4(1-0.5)}[(0-\alpha_4^\beta)^+ + (0-\alpha_4^\beta)^+ + (10-\alpha_4^\beta)^+ + (20-\alpha_4^\beta)^+]]$$

$$= \underset{\alpha_4^\beta}{Min}\ [\alpha_4^\beta + (0-\alpha_4^\beta)^+ + \frac{1}{2}(10-\alpha_4^\beta)^+ + \frac{1}{2}(20-\alpha_4^\beta)^+]$$

Then after minimizing ϕ_4^β we have: ϕ_4^β=15, and $0 \le \alpha_4^\beta \le 10$. Next we calculate ϕ_5^β.

$$\phi_5^\beta = \underset{\alpha_5^\beta}{Min}\ [\alpha_5^\beta + \frac{1}{4(1-0.5)}[(0-\alpha_5^\beta)^+ + (0-\alpha_5^\beta)^+ + (80-\alpha_5^\beta)^+ + (100-\alpha_4^\beta)^+]]$$

$$= \underset{\alpha_5^\beta}{Min}\ [\alpha_5^\beta + (0-\alpha_5^\beta)^+ + \frac{1}{2}(80-\alpha_5^\beta)^+ + \frac{1}{2}(100-\alpha_5^\beta)^+]$$

Finally, by minimizing ϕ_5^β we have: ϕ_5^β=90, and $0 \le \alpha_5^\beta \le 80$. Next, we calculate the CVaR at the root node i.e., ϕ_1^β.

$$\phi_1^\beta = \underset{\alpha_1^\beta}{Min}\ [\alpha_1^\beta + \frac{1}{4(1-0.5)}[(0-\alpha_1^\beta)^+ + (10-\alpha_1^\beta)^+ + (20-\alpha_1^\beta)^+ + (80-\alpha_1^\beta)^+]$$
$$+ \frac{1}{4}[0+5+15+90]]$$

$$= \underset{\alpha_1^\beta}{Min}\ [\alpha_1^\beta + \frac{1}{2}(0-\alpha_1^\beta)^+ + \frac{1}{2}(10-\alpha_1^\beta)^+ + \frac{1}{2}(20-\alpha_1^\beta)^+ + \frac{1}{2}(80-\alpha_1^\beta)^+ + 27.5] \quad (4.21)$$

We have represented the equation (4.21) in Figure 4.4. As seen from Figure 4.4, the minimum values of ϕ_1^β, and α_1^β are 77.5 and 10, respectively.

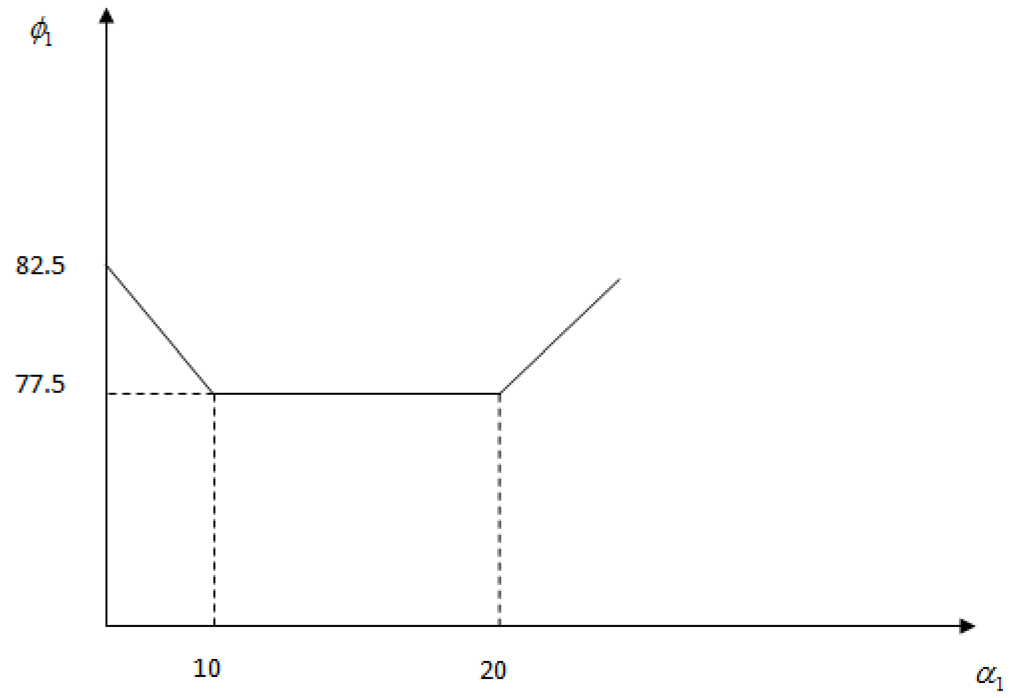

Figure 4.4: Illustration of equation (4.21), with β level of 50% and S_t= 4

Next, we consider equation (4.20). This formula is adapted from Shapiro (2011). He has not considered the value of CVaRs at the child nodes. Indeed, he has used the concept of cost-to-go functions in the time consistent definition of CVaR. So, we proceed by calculation of Q_m, using equation (4.13), at the nodes 2, 3, 4, and 5. Obviously the value of Q_m at the final nodes is zero.

$$Q_2 = L_2 + \frac{1}{4}(0+0+0+0) = 0$$

$$Q_3 = L_3 + \frac{1}{4}(0+0+0+10) = 12.5$$

$$Q_4 = L_4 + \frac{1}{4}(0+0+0+10) = 27.5$$

$$Q_5 = L_5 + \frac{1}{4}(20+40+80+100) = 140$$

Now, we proceed for calculation of CVaR at the root node using equation (4.20).

$$\phi_1^{\beta} = \underset{\alpha_1^{\beta}}{Min}\ [\alpha_1^{\beta} + \frac{1}{4(1-0.5)}[(0-\alpha_1^{\beta})^+ + (12.5-\alpha_1^{\beta})^+ + (27.5-\alpha_1^{\beta})^+ + (140-\alpha_1^{\beta})^+]]$$

(4.22)

We have represented the equation (4.22) in Figure 4.5. As seen from Figure 4.5, the minimum values of ϕ_1^{β}, and α_1^{β} are 83.75 and 12.5, respectively.

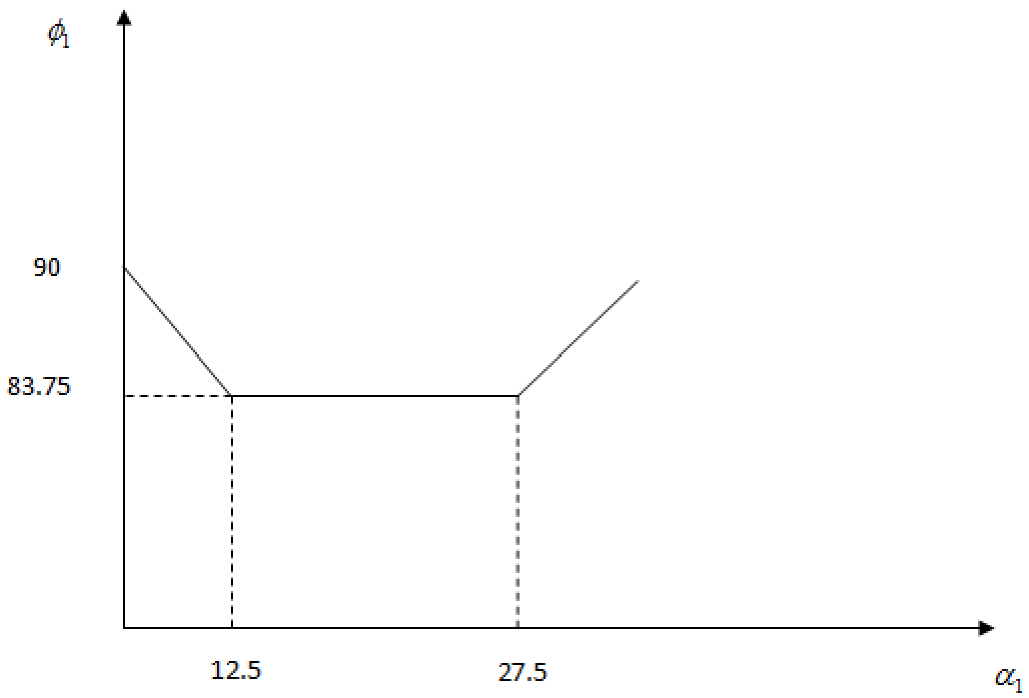

Figure 4.5: Illustration of equation (4.22), with β level of 50% and S_t= 4

In order to understand better the dynamic formulation of CVaR in (4.20), adapted from Shapiro (2011), we represent a different numerical example and we compare the results with RECVaR in equation (4.19). We have changed the losses at the final nodes with higher dispersion in the previous example shown by Figure 4.6. But, the expected losses at the final nodes are the same in Figure 4.3.

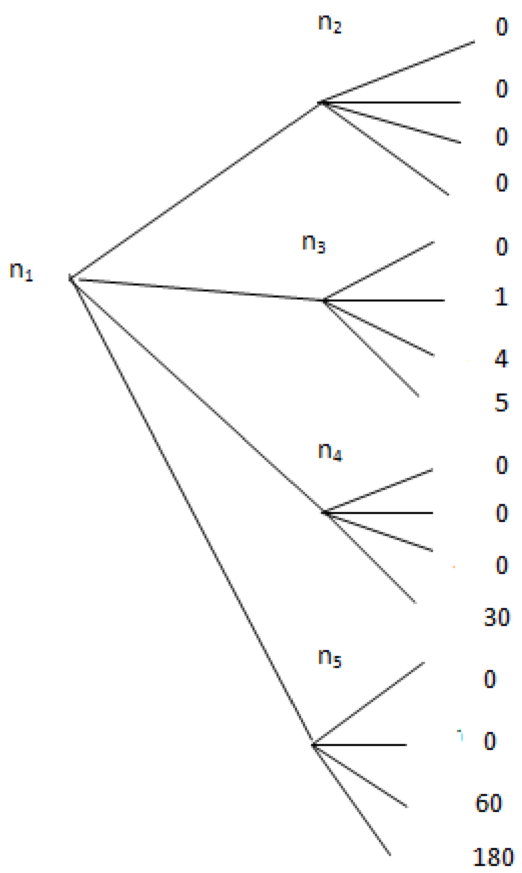

Figure 4.6: A three stage scenario tree with S_t= 4 at each stage with different losses at final nodes

The new CVaRs at stage 2, which are the nodes 2, 3, 4, and 5, i.e, $\phi_2^\beta, \phi_3^\beta, \phi_4^\beta$, and ϕ_5^β are 0, 4.5, 15, and 120, respectively. So, we have:

By using equation (4.19):

$$\phi_1^\beta = \underset{\alpha_1^\beta}{Min}\ [\alpha_1^\beta + \frac{1}{4(1-0.5)}[(0-\alpha_1^\beta)^+ + (10-\alpha_1^\beta)^+ + (20-\alpha_1^\beta)^+ + (80-\alpha_1^\beta)^+]$$
$$+\frac{1}{4}[0+4.5+15+120]]$$

If we minimize ϕ_1^β, we obtain ϕ_1^β=84.875 and α_1^β=10.

For the equation (4.20), we proceed by calculation of Q_m, using equation (4.13), at nodes 2, 3, 4, and 5, with new losses at the final nodes in Figure 4.6.

$$Q_2 = L_2 + \frac{1}{4}(0+0+0+0) = 0$$
$$Q_3 = L_3 + \frac{1}{4}(0+1+4+5) = 12.5$$
$$Q_4 = L_4 + \frac{1}{4}(0+0+0+30) = 27.5$$
$$Q_5 = L_5 + \frac{1}{4}(0+0+60+180) = 140$$

By using equation (4.20):

$$\phi_1^\beta = \underset{\alpha_1^\beta}{Min}\ [\alpha_1^\beta + \frac{1}{4(1-0.5)}[(0-\alpha_1^\beta)^+ + (12.5-\alpha_1^\beta)^+ + (27.5-\alpha_1^\beta)^+ + (140-\alpha_1^\beta)^+]]$$

If we minimize ϕ_1^β, we obtain ϕ_1^β=83.75 and α_1^β=12.5.

Because the losses at the final nodes have higher dispersion, the CVaR at the root node using equation (4.19) is higher than previous case. But, using (4.20) gives us the same result as before despite the fact that the CVaRs in the middle nodes i.e., ϕ_2^β, ϕ_3^β, ϕ_4^β, and ϕ_5^β have been changed. So, using Shapiro (2011) formulation for time consistent version of CVaR does not consider the impact of risk at the child nodes. As a result, this is the main critique of the time consistent formulation of CVaR suggested by Shapiro (2011).

So, we can conclude that using different dynamic risk measures in which both of them are time consistent *by construction*, we have different results. Now, the question is which one we should use? The answer is the one which is also coherent.

We have already defined the properties of a coherent risk measure in Chapter 2 through equations (2.32) to (2.34) which are Positive Homogeneity, Translation Invariance, and convexity (Artzner et al., 1999). In the next section we consider the coherency property of the two defined risk measures and the effect of clustering on the CVaR.

4.3 Analyzing the Main Properties of the Model

First, we prove that equation (4.19) is a coherent risk measure. Indeed, we want show that the dynamic risk measure ϕ_n^β in (4.19) satisfies all properties of a coherent risk measure.

Proposition 4.1: *The dynamic recursive risk measure in* (*4.19*) *is coherent.*

Proof: First, we consider the Positive Homogeneity property. By using (4.19), $\phi_n^\beta(L) = \underset{\alpha_n^\beta}{Min}\left[\alpha_n^\beta(L) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)} z_m(L) + \frac{1}{S_t}\sum_{\psi(n,m)}\phi_m(L)\right]$, we have:

$$\phi_n^\beta(hL) = \underset{\alpha_n^\beta}{Min}\begin{bmatrix} (\alpha_n^\beta(hL) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(hL_m - \alpha_n^\beta(hL))^+ \\ +\dfrac{1}{S_t}\displaystyle\sum_{\psi(n,m)}\left[\alpha_m^\beta(hL) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(hL_{mm} - \alpha_m^\beta(hL))^+\right]\end{bmatrix} \tag{4.23}$$

In (4.23), L_{mm} is the loss function at child of child m. Because VaR has Positive Homogeneity property we can write (4.23) as:

$$= h\underset{\alpha_n^\beta}{Min}\begin{bmatrix} \alpha_n^\beta(L) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_m - \alpha_n^\beta(L))^+ \\ +\dfrac{1}{S_t}\displaystyle\sum_{\psi(n,m)}\left[\alpha_m^\beta(L) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_{mm} - \alpha_m^\beta(L))^+\right]\end{bmatrix} \tag{4.24}$$

and it follows :

$$\phi_n^{\beta}(hL) = h\phi_n^{\beta}(L)$$

Next, for proving Translation Invariance property we have:

$$\phi_n^{\beta}(L+h) = \underset{\alpha_n^{\beta}}{Min}\left[\begin{array}{l}\alpha_n^{\beta}(L+h) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_m + h - \alpha_n^{\beta}(L+h))^{+} \\ +\dfrac{1}{S_t}\displaystyle\sum_{\psi(n,m)}\left[\alpha_m^{\beta}(L+h) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_{mm} + h - \alpha_m^{\beta}(L+h))^{+}\right]\end{array}\right]$$

(4.25)

Because VaR has Translation Invariance property we can write (4.25) as:

$$= \underset{\alpha_n^{\beta}}{Min}\left[\begin{array}{l}\alpha_n^{\beta}(L) + h + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_m + h - \alpha_n^{\beta}(L) - h)^{+} \\ +\dfrac{1}{S_t}\displaystyle\sum_{\psi(n,m)}\left[\alpha_m^{\beta}(L) + h + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_{mm} + h - \alpha_m^{\beta}(L) - h)^{+}\right]\end{array}\right] \qquad (4.26)$$

And it follows:

$$\phi_n^{\beta}(L+h) = \phi_n^{\beta}(L) + 2h$$

And if we consider until the last stage *n*, we have:

$$\phi_n^{\beta}(L+h) = \phi_n^{\beta}(L) + nh \qquad (4.27)$$

In (4.27), we can say that by adding *h* through the stages (time periods), the risk will be higher. Indeed, it increases uncertainty about the future. We can call it time-adjusted Translation Invariance property. So, the equation in (4.19) satisfies Translation Invariance property.

Next, for proving convexity property, we use discrete static formulation of CVaR, (Rockafellar and Uryasev, 2000), in (4.28).

$$\phi^{\beta}(\mathrm{X}_i) = \underset{\alpha_i^{\beta}}{Min}\left[\alpha_i^{\beta} + \frac{1}{1-\beta}E[\mathrm{X}_i - \alpha_i^{\beta}]^{+}\right] \tag{4.28}$$

Let us $f(\mathrm{x}) = [x-\alpha]^{+}$. Because, $f(\mathrm{x})$ is a convex function we use it for proving the convexity of CVaR in (4.28) and we can write its definition by:

$$\forall x_1, x_2 \in [a,b],\ \exists \lambda \in [0,1],\ f(\lambda x_1 + (1-\lambda)x_2) \le \lambda f(x_1) + (1-\lambda) f(x_2)$$

By replacing the $[x-\alpha]^{+}$ instead of $f(\mathrm{x})$ we have:

$$[\lambda x_1 + (1-\lambda)x_2 - \alpha]^{+} \le \lambda[x_1 - \alpha]^{+} + (1-\lambda)[x_2 - \alpha]^{+} \tag{4.29}$$

We can use equation in (4.29) for proving the convexity of discrete static formulation of CVaR, (Rockafellar and Uryasev, 2000), in (4.28).

$$\phi^{\beta}(\lambda X_1 + (1-\lambda)X_2) =$$
$$\underset{\lambda\alpha_1^{\beta}+(1-\lambda)\alpha_2^{\beta}}{Min}\left[\lambda\alpha_1^{\beta} + (1-\lambda)\alpha_2^{\beta} + \frac{1}{1-\beta}E[\lambda X_1 + (1-\lambda)X_2 - \lambda\alpha_1^{\beta} - (1-\lambda)\alpha_2^{\beta}]^{+}\right] \tag{4.30}$$

By using property of convexity of $f(x)$ in (4.29) we have:

$$\lambda\alpha_1^{\beta} + (1-\lambda)\alpha_2^{\beta} + \frac{1}{1-\beta}E[\lambda X_1 + (1-\lambda)X_2 - \lambda\alpha_1^{\beta} - (1-\lambda)\alpha_2^{\beta}]^{+}$$

$$\le \lambda\alpha_1^{\beta} + (1-\lambda)\alpha_2^{\beta} + \frac{\lambda}{1-\beta}E[X_1 - \alpha_1^{\beta}]^{+} + \frac{1-\lambda}{1-\beta}E[X_2 - \alpha_2^{\beta}]^{+} \tag{4.31}$$

And by sorting the terms in (4.31) we have:

$$\lambda\alpha_1^\beta + (1-\lambda)\alpha_2^\beta + \frac{\lambda}{1-\beta}E[X_1 - \alpha_1^\beta]^+ + \frac{1-\lambda}{1-\beta}E[X_2 - \alpha_2^\beta]^+$$

$$\leq \lambda(\alpha_1^\beta + \frac{1}{1-\beta}E[X_1 - \alpha_1^\beta]^+) + (1-\lambda)(\alpha_2^\beta + \frac{1}{1-\beta}E[X_2 - \alpha_2^\beta]^+)$$

And finally, by using definition of static CVaR in equation (4.28) we have:

$$\lambda(\alpha_1^\beta + \frac{1}{1-\beta}E[X_1 - \alpha_1^\beta]^+) + (1-\lambda)(\alpha_2^\beta + \frac{1}{1-\beta}E[X_2 - \alpha_2^\beta]^+)$$

$$\leq \lambda\phi^\beta(X_1) + (1-\lambda)\phi^\beta(X_2) \qquad (4.32)$$

Equation (4.32) clearly shows that CVaR satisfies convexity property. As a result, the static part in equation (4.19) is coherent. Now, for the dynamic part, $E(\sum_{\psi(n,m)} \phi_m) = \frac{1}{S_t}\sum_{\psi(n,m)} \phi_m$, we can say that, it is the definition of static CVaR at child node *m*. Because, we have proved already the static CVaR in parent node *n* convex, we can conclude that static CVaR at child *m*, inherits the convexity from its parent node. As a result, the equation in the dynamic part is also convex and the summation of two convex functions is also convex. ■

Next, we consider the coherency property of the dynamic risk measure in equation (4.20).

Proposition 4.2: *The dynamic recursive risk measure in (4.20) is coherent.*

Proof: First, we consider the Positive Homogeneity property. By using (4.20), $\phi_n^\beta(L) = \underset{\alpha_n^\beta}{Min}\left[\alpha_n^\beta(L) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)}(Q_m(L_m) - \alpha_n^\beta(L))^+\right]$, we have:

$$\phi_n^\beta(hL) = \underset{\alpha_n^\beta}{Min}\left[\alpha_n^\beta(hL) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)}(Q_m(hL_m) - \alpha_n^\beta(hL))^+\right]$$

And using the definition of Q_m in (4.13),

$$= \underset{\alpha_n^\beta}{Min}\left[\alpha_n^\beta(hL) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)}\left[hL_m + \frac{1}{S_t}\sum_{\psi(n,m)}Q_{mm}(hL_m) - \alpha_n^\beta(hL)\right]^+\right] \qquad (4.33)$$

In (4.33), Q_{mm} is the expected cost function at child node of child *m*. Because VaR has Positive Homogeneity property we can write (4.33) as:

$$= h\underset{\alpha_n^\beta}{Min}\left[\alpha_n^\beta(L) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)}\left[L_m + \frac{1}{S_t}\sum_{\psi(n,m)}Q_{mm}(L_m) - \alpha_n^\beta(L)\right]^+\right] \qquad (4.34)$$

And it follows:

$$\phi_n^\beta(hL) = h\phi_n^\beta(L)$$

Next, for proving Translation Invariance property we have:

$$\phi_n^\beta(L+h) = \underset{\alpha_n^\beta}{Min}\left[\alpha_n^\beta(L+h) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)}(Q_m(L_m+h) - \alpha_n^\beta(L+h))^+\right]$$

And using definition of Q_m in (4.13),

$$= \underset{\alpha_n^\beta}{Min}\left[\alpha_n^\beta(L+h) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)}\left[L_m + h + \frac{1}{S_t}\sum_{\psi(n,m)}Q_{mm}(L_m+h) - \alpha_n^\beta(L+h)\right]^+\right]$$

(4.35)

Because VaR has Translation Invariance property, we can write (4.35) as:

$$= \underset{\alpha_n^{\beta}}{Min}\left[\alpha_n^{\beta}(L)+h+\frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)}\left[L_m+h+\frac{1}{S_t}\sum_{\psi(n,m)}Q_{mm}(L_m+h)-\alpha_n^{\beta}(L)-h\right]^+\right]$$

And it follows:

$$\phi_n^{\beta}(L+h)=\phi_n^{\beta}(L)+2h$$

And if we consider until the last stage *n*, we have:

$$\phi_n^{\beta}(L+h)=\phi_n^{\beta}(L)+nh \tag{4.36}$$

Like the proof in Proposition 4.1, we can name equation (4.36) as time-adjusted Translation Invariance property. So, the equation in (4.20) satisfies Translation Invariance property.

For the proof of convexity property of equation (4.20), we use the same procedure in proposition 4.1. As a result, the risk measure in (4.20) satisfies all properties of a coherent risk measure. ■

So far in this Chapter, we have introduced two different risk measures which both of them are coherent and time consistent. We have also presented the drawback of risk measure adapted from Shapiro (2011) by a numerical example and we have found that the dynamic risk measure is not sensitive to the values of the CVaR in the middle stages of the scenario tree. So, we prefer to use first recursive risk measure which provides more intuitive results for considering the risk minimization in the objective function.

Next, we consider the effect of clustering on CVaR. In Proposition 4.3, we use the static definition of CVaR, Rockafellar and Uryasev (2000), for considering the portfolio effect of clustering.

Proposition 4.3:. *Let* $E(\phi_k^\beta)$ *represent the expected CVaR for each one of k clusters, and* ϕ^β *denote the CVaR for the combined clusters. Then,*

$$E(\phi_k^\beta) \le \phi^\beta \le \sum_k \phi_k^\beta \quad . \tag{4.37}$$

Proof: Let S_k represent the number of observations in cluster k and $S_1 + S_2 + ... + S_k = S$ in which S is the total number of observations in the combined set. By using static definition of CVaR, Rockafellar and Uryasev (2000), we have

$$\phi^\beta = \underset{\alpha^\beta}{Min}\left[\alpha^\beta + \frac{1}{(1-\beta)S}\sum_{j\in S}(L_j - \alpha^\beta)^+\right] \tag{4.38}$$

in which L_j represents the loss function associated with observation j which belongs to set S.

The upper limit of ϕ^β in inequality (4.37) can be obtained directly from subadditivity property of CVaR (Artzner et al., 1999). Then, for the lower limit

$$E(\phi_k^\beta) = \frac{S_1\phi_1^\beta + S_2\phi_2^\beta + ... + S_k\phi_k^\beta}{S_1 + S_2 + ... + S_k} .$$

Hence, By using equation (4.38):

$$E(\phi_k^\beta) = \frac{S_1\alpha_1^\beta + \frac{1}{1-\beta}\sum_{j\in s_1}(L_j - \alpha_1^\beta)^+ + ... + S_k\alpha_k^\beta + \frac{1}{1-\beta}\sum_{j\in s_k}(L_j - \alpha_k^\beta)^+}{S_1 + S_2 + ... + S_k}$$

$$= \frac{S_1\alpha_1^\beta + S_2\alpha_2^\beta + ... + S_2\alpha_2^\beta}{S} + \frac{1}{(1-\beta)S}\left[\sum_{j\in s_1}(L_j - \alpha_1^\beta)^+ + ... + \sum_{j\in s_k}(L_j - \alpha_k^\beta)^+\right]$$

from which we derive

$$E(\phi_k^\beta) = E_k(\alpha_k^\beta) + \frac{1}{(1-\beta)S}\left[\sum_{j\in s_1}(L_j - \alpha_1^\beta)^+ + \ldots + \sum_{j\in s_k}(L_j - \alpha_k^\beta)^+\right]. \tag{4.39}$$

Now, we consider two separate cases: (a) $\forall k,\ \alpha_k^\beta = \alpha^\beta$, which means that, with clustering the decisions are the same in all the clusters and that the VaR doses not change. As a result, the α_k^β for all the clusters is equal to the α^β of the combined set. Then equation (4.39) can be represented as

$$E(\phi_k^\beta) = \alpha^\beta + \frac{1}{(1-\beta)S}\sum_{j\in S}(L_j - \alpha^\beta)^+ \to E_k(\phi_k^\beta) = \phi^\beta .$$

(b) $\forall k,\ \alpha_k^\beta \neq \alpha^\beta$, in this case, by clustering we gain more degrees of freedom; therefore, by optimality the loss (cost) at every observation, in each cluster, is less or equal to the cost in the combined cluster. Therefore, α_k^β, in each cluster, is less than or equal to the α^β in the combined cluster and, consequently, the expected value of the CVaR of the clusters is less than CVaR for the combined set. Indeed, if we compare equation (4.38) with (4.39), each term in (4.39) is less than the corresponding term in (4.38). As a result, $E(\phi_k^\beta) < \phi^\beta$. By taking into account both cases of (a) and (b) the inequality (4.37) is proved. ■

4.4 A Case Study on Sustainable Fleet Replacement

4.4.1 Definition of the Parameters

We use the historical data for fuel prices from Jan. 2000 to Dec. 2012. As can be seen from Table 4.2, when we consider the average correlation coefficient in each year between fuel prices from Jan. 2000 to Dec. 2012, the diesel and petrol prices have a very high correlation coefficient of 0.876 but a negative correlation of (-0.126) and (-0.162) with electricity, respectively (in this thesis, we use electricity and electricity charge prices interchangeably). However, the price of electricity is the price of each charge for a 22 kwh battery. This result suggests that, in the short-term, we can use electric vehicles to hedge the risk of fuel price rises but that petrol and diesel vehicles, due to the high correlation, are exposed to the same sources of risk and are not very useful for reducing each other's risk exposure.

Based on this observation, we can generate the simulated scenarios for fuel prices with the above correlation matrix using the expected forecasted prices from 2013 to the end of 2020 (Table 4.3). Next, we consider the different approaches for the clustering analysis and then use the one most appropriate for our data.

In this Chapter, the following convention is used to denote different vehicle types and engine technologies. Internal combustion engine vehicles, also called conventional vehicles, use gasoline or a fossil fuel as the only source of energy. Hybrid vehicles have an internal combustion engine but also a battery that can be used to power the vehicle wheels. Finally, electric vehicles only have an electric engine and no combustion engine.

Table 4.2: Correlation matrix for fuel prices from Jan. 2000 to Dec. 2012 in each year

	Petrol	Diesel	Electricity
Petrol	1	0.876	-0.126
Diesel	0.876	1	-0.162
Electricity	-0.126	0.162	1

Table 4.3: Average forecasted fuel prices from 2013 to 2020

	2013	2014	2015	2016	2017	2018	2019	2020
Petrol	1.338	1.386	1.435	1.483	1.532	1.58	1.628	1.677
Diesel	1.387	1.439	1.49	1.542	1.593	1.645	1.696	1.748
Electricity	2.476	2.581	2.685	2.789	2.894	2.998	3.102	3.206

At each stage (year), we want to have the minimum number of vehicles in the fleet system, taking into account that the contract of the vehicles will be retired by the end of the fourth year. Moreover, we assume an initial condition of the 2013 fleet system consisting of 2369 diesel vehicles with different capacities (small, light, medium), which are distributed with a percentage age vector [22%, 40%, 30%, 8%] for the corresponding age of the vehicles in years from one to four. All the vehicles are assumed to be vans. Obviously, the vehicles with their contract in the fourth year will be retired the following year. We also consider four technologies: petrol, hybrid-petrol, hybrid-diesel, and EVs. We assume that other technologies have different capacities (small, light, and medium), such as diesel vans. The CO_2 emissions for petrol, diesel, hybrid-petrol and hybrid-diesel are 2310, 2680, 1719, and 2177 (g/litre), respectively. Moreover, the CO_2 emissions for EVs are 81 g/km. The leasing costs for each capacity and technology are summarised in Table 4.4.

Table 4.4: The leasing costs (£) for vehicles with different capacities and technologies

Technology	Small (£)	Light(£)	Medium(£)
Petrol	209	228	266
Diesel	220	240	280
Hybrid-Petrol	299	326	381
Hybrid-Diesel	319	348	406
EVs	381	415	484

For the petrol and hybrid technologies, we use their expected values of fuel consumption as a base in each cluster for diesel technology. By multiplying the fuel consumption coefficient vector, [1.31, 1, 0.93, 0.8], we obtain the corresponding fuel consumption of petrol, diesel, petrol-hybrid, and diesel-hybrid for each cluster, respectively. The benchmark is indicated by the number 1 and corresponds to diesel technology. For EVs we take into account the forecasted electricity prices shown in Table 4.3 for small capacity as a base and by multiplying the ratio of power of batteries, [1, 1.18, 1.77], for small, light and medium, respectively, by which we can obtain the electricity prices for each cluster in different capacities of EVs. The benchmark is the first component of the vector and is shown by 1. Furthermore, for the monthly lease cost of batteries for EVs we assume the base price for small capacity (22 kw) is £79, and for obtaining the monthly cost of batteries for other capacities we multiply by the same ratio vector [1, 1.18, 1.77].

4.4.2 Using Clustering Analysis

The aim of the clustering analysis is to identify homogenous subgroups of instances in a population. For large numbers of observations, hierarchical clustering algorithms can be time consuming (e.g., Schonlau, 2004). The computational complexity of the three popular agglomerative hierarchical methods (single, complete and average linkage) is of the order O(n^2), whereas the most popular non-hierarchical cluster algorithm, *K*-means (MacQueen, 1967), is only of the order O(*Kn*), where *K* is the number of clusters and *n* the number of observations (Hand et al., 2001). Therefore, *K*-means, a non-hierarchical method, is emerging as a popular choice in the data mining community. We implement a two-step clustering algorithm that is well-suited when we address a large dataset, and it combines the abilities of hierarchical clustering analysis and *K*-means simultaneously. A two-step cluster analysis is conducted using the Bayesian Information Criterion to determine the number of clusters automatically (Bacher et al., 2004).

We consider three vehicle types with different capacities. Capacity is an important characteristic because, depending on the function, the technicians in the company have to carry materials and, thus, have to drive a vehicle with sufficient capacity. For instance, power engineers need light vans with enough carrying capacity. We consider small vans as weighing 300 kg and that light vans weigh 500 kg and have a greater carrying capacity. Medium vans can be used on any type of function, but there are only a few because they are far more expensive to buy and maintain. We perform cluster analysis for each type of vehicle. The total number of vans is 2369, comprising 1137 (48%) small, 1077 (45%) light, and 155 (7%) medium vans.

The result of clustering analysis for the small vans is presented in Table 4.5. The clusters are sorted by an increasing order of mileage driven. The high mileage driven vehicles for small vans are distributed among cluster 4 with normal fuel efficiency. In this cluster, we have 324 vehicles (28%). There are two other clusters, clusters 2 and 3, which are below the average mileage driven (1011 miles/month), which have similar mileages, but have different fuel consumptions. Finally, in cluster 1 we have the lowest mileage vehicles (581 miles/month), with less efficient fuel consumption: the vehicles in this group are underutilised and are of great concern for the fleet management.

Table 4.5: Cluster analysis for mileage and fuel consumption data for small vans

Types of vehicles	Cluster	Monthly Mileage (miles)		Fuel Consumption (litres/100km)		Vehicles	
		Mean	S.D.	Mean	S.D.	Number	%
Small Vans	1	581.4	267.8	6.3	0.4	135	12%
	2	789.2	254.9	5.3	0.3	381	34%
	3	845.6	255.7	4.5	0.4	297	26%
	4	1605.0	313.6	5.0	0.4	324	28%
	Combined	1011.7	470.5	5.1	0.7	1137	100%

For the light vans (Table 4.6), the vehicles that can be classified as running a higher mileage are distributed in two clusters, 4 and 5, with different fuel efficiencies. In cluster 5 we have 142 vehicles (13%), which have a mileage of 1924.2 miles/month and have a fuel consumption of 5.75 litres/100 km. In contrast, in cluster 4 we have 189 vehicles (18%) with a mileage of 1376 miles/month and a fuel consumption of 6.63 litres/100 km. In cluster 3, we have the largest portion of the light vans, 324 vehicles (30%), which can be classified as average mileage vehicles, though with an efficient fuel consumption of 5.38 litres/100 km. We also have two clusters for lower mileage vehicles, clusters 1 and 2, which are very close to each other in terms of mileage driven but are not the same in terms of fuel consumption.

For the medium vans (Table 4.7), the underutilised segment (i.e., cluster 1 of medium vans) has a percentage of 27% of the vehicles, with the least amount of mileage driven in a month equal to 735.39 miles and the fuel consumption equal to 10.37 litres/100 km. In addition, the largest portion of the medium vans belongs to normal mileage driven vehicles, cluster 2, with an average of 964.73 miles/month and a very efficient fuel consumption of 8.01 litres/100 km. Finally, the third segment for high mileage driven vehicles has a contribution of 29% to the whole number of medium vans, with an average mileage of 1617.97 miles/month and a close to normal fuel consumption of 9.39 litres/100 km.

Table 4.6: Cluster analysis for mileage and fuel consumption data for light vans

Types of vehicles	Cluster	Monthly Mileage (miles)		Fuel Consumption (litres/100km)		Vehicles	
		Mean	S.D.	Mean	S.D.	Number	%
Light Vans	1	635.7	241.7	7.1	0.5	130	12%
	2	721.0	206.1	6.0	0.3	292	27%
	3	1110.4	253.8	5.4	0.5	324	30%
	4	1376.8	233.7	6.6	0.5	189	18%
	5	1924.2	313.0	5.8	0.4	142	13%
	Combined	1101.0	477.9	6.0	0.7	1077	100%

Table 4.7: Cluster analysis for mileage and fuel consumption data for medium vans

Types of vehicles	Cluster	Monthly Mileage (miles)		Fuel Consumption (litres/100km)		Vehicles	
		Mean	S.D.	Mean	S.D.	Number	%
Medium Vans	1	735.4	245.3	10.4	1.1	42	27%
	2	964.7	193.0	8.0	1.2	68	44%
	3	1618.0	297.2	9.4	1.3	45	29%
	Combined	1092.2	424.4	9.1	1.6	155	100%

4.5 Solving the Fleet Replacement Problem

Given these constraints, we want to minimise the weighted average of total expected costs and CVaR during the planning horizon. Our goal is to determine the optimal policy from 2014 to 2017. Note that in order to calculate this policy we need to continue the calculation for the stochastic variables from 2017 to the end of 2020 (until the end of life of the vehicles leased in the period of analysis).

Moreover, at each node we have generated a random vector of normal distribution for fuel prices, monthly mileage driven, and fuel consumption. For the means and standard deviations of corresponding distributions, we have used the information in Tables 4.5, 4.6, and 4.7. Moreover, we can use a normal distribution for fuel consumption and mileage driven in each cluster. For simulating the CO_2 prices, we have assumed a uniform distribution between £5/ton and £20/ton.

The number of vehicles at each node follows a normal distribution with the mean equal to the size of each cluster and a standard deviation equal to 5% of the corresponding size of the cluster. The reason for this assumption is twofold: first, the number of vehicles (demand) in each year is a stochastic parameter; second, the distribution of leased vehicles needed in each year based on the real data follows a normal distribution in which the variation of vehicles is 5% of the total fleet size.

To consider the effect of the branching level (S_t) corresponding to each parent node at each stage t, we use fives cases in which we have different branching levels in early stages focusing on the first stage. This is because the uncertainties by closing to the final stages become lower due to a greater realisation of stochastic processes. In the last row of Table 4.8, we show the percentage of difference between the corresponding CVaR, in each case, and the previous one. For example, the percentage of difference between the CVaR in case 2 and in case 1 is 1.47%. By increasing the number of scenarios, the difference is reduced and the CVaR converges. Thus, we can conclude that we have the convergence of CVaR for approximately 24000 scenarios with a tolerance of 0.1%. Based on this observation, we run the simulations based on the pattern in case number 4 and we can be confident about the accuracy of the results.

Table 4.8: Effect of branching (S_t) on the convergence of CVaR.

Case/year	1	2	3	4	5
S_1	10	15	20	30	40
S_2	10	10	10	10	10
S_3	5	5	5	5	5
S_4	3	2	2	2	2
S_5	2	2	2	2	2
S_6	2	2	2	2	2
S_7	2	2	2	2	2
S_8	2	2	2	2	2
# nodes	4711	23416	31221	46831	62441
# scenarios	2400	12000	16000	24000	32000
CVaR difference(%)	-	1.47%	0.43%	0.37%	0.1%

We have considered different values of ω (The parameter that models the trade-off between risk and expected cost). Moreover, to obtain the optimal number of vehicles at each cluster we take into account the corresponding number of vehicles in each type of the vehicles, from Tables 4.4, 4.5, and 4.6. Next, we proceed by considering each type of vehicle separately.

4.5.1 Result of Optimal Policies for Technologies in Different Types of Vehicles

The results for each vehicle type are shown in Table 4.9. We considered three values of ω equal to 0, 0.5, and 1. Moreover, we calculated the average number of new leased vehicles at different nodes in each year (stage). We denote different technologies by D, P, H-P, H-D, and E for Diesel, Petrol, Hybrid-Petrol, Hybrid-Diesel, and Electric, respectively. Based on different risk preference parameter (ω) values, the results indicate that we should lease a different number of vehicles at different stages. For example, as seen in Table 4.9 (a), which shows the results for small vans, in cluster 1, for a ω equal to 0.5, we should lease 11, 38, 4, and 0 diesel vehicles; 0, 2, 50, and 30 petrol vehicles at the beginning of each year, between 2014 and 2017, respectively. Indeed, petrol vehicles have a high percentage in lower mileage clusters when ω equal to 0.5. For example in cluster 1 of small vans for ω equal to 0.5, we should lease 61% of the whole vehicles with petrol vehicles. In addition, for clusters 2 and 3 at the beginning of 2017 for ω equal to 0, we should lease also hybrid-petrol and hybrid-diesel with a small numbers. For cluster 4, which has vehicles with higher mileage profiles, for ω equal to 0 and 0.5, we should consider leasing hybrid-petrol and hybrid-diesel and electric vehicles with a small penetration ratio. For example, for cluster 4 of small vans during 2014 to 2017 for ω equal to 0, the percentage of new leased diesel, petrol, hybrid-petrol, hybrid-diesel, and EVs are 89%, 4%., 3%, 2%, and 2% , respectively.

We now analyse the results for light vans. As we have seen in Section 4.3 for light vans there are five clusters of which the first two clusters are considered for lower mileage vehicles, with different fuel efficiencies, and the third cluster is for normal mileage vehicles. Moreover, the last two clusters, i.e., clusters 4 and 5, include higher mileage vehicles with different fuel consumptions. Given different values of ω equal to 0, 0.5, and 1, the optimal choice of technology, and the number of new leased vehicles, Table 4.9 (b), for all clusters are obtained. In light vans we have also a combination of different technologies with different penetration ratios in different clusters. But, in higher mileage clusters the penetration ratio of EVs is higher. For example, in cluster 5 of light vans during 2014 to 2107, for ω equal to 0, the penetration ratio of diesel, petrol, hybrid-petrol, hybrid-diesel, and EVs are 81%, 1%, 2%, 3%, and 13%. However, by increasing the value of ω, this ratio decreases to 4% and 2% for a ω equal to 0.5 and 1 for EVs, respectively.

Next, we consider the case study results for medium vans, which have the smallest portion (7%) of the total numbers of vehicles due to their weight, cost and special tasks, which are assigned to them for specific cases. As we discussed before, three clusters for these vehicle types have been identified. The model results for the clusters are presented in Table 4.9 (c). Given different values of ω, the optimal choice of technologies for clusters 1 and 2 is Diesel, Petrol, Hybrid-Petrol, and Hybrid-Diesel. For the third cluster, which is considered for high mileage vehicles, the optimal choice also includes EVs. Due to higher investment costs compared with small and light vans, the electric technology is the optimal choice for cluster 3 of medium vans, with high penetration ratios of 56%, 24%, and 11%, for a ω equal to 0, 0.5, and 1, respectively.

Table 4.9: Optimal number of vehicles per cluster. For each cluster, and ω equal to 0, 0.5 and 1, we report the technologies leased and the number of vehicles per technology.

(a) Small Vans

		Cluster 1			Cluster 2		
ω		0	0.5	1	0	0.5	1
Year	2014	34	11, 0	11	80, 0, 0, 0	30, 0	30
	2015	44	38, 2	41	125, 0, 0, 0	115, 0	115
	2016	44	4, 50	54	101, 27, 0, 0	14, 138	152
	2017	13	0, 30	29	26, 16, 3, 3	3, 81	84
Tech.		D	D, P	D	D, P, HP, HD	D, P	D
		Cluster 3			**Cluster 4**		
ω		0	0.5	1	0	0.5	1
Year	2014	60, 0, 0, 0	23, 1	24	73, 0, 1, 0, 0	25, 1, 0, 0, 0	26
	2015	94, 0, 0, 0	83, 6	89	97, 0, 0, 0, 0	99, 0, 0, 0, 0	97
	2016	78, 26, 0, 0	8, 111	119	98, 8, 4, 3, 2	84, 42, 1, 1, 1	130
	2017	20, 14, 3, 2	2, 63	65	20, 6, 4, 4, 4	22, 44, 1, 2, 1	71
Tech.		D, P, HP, HD	D, P	D	D, P, HP, HD, E	D, P, HP, HD, E	D

(b) Light Vans

		Cluster 1			Cluster 2			Cluster 3		
ω		0	0.5	1	0	0.5	1	0	0.5	1
Year	2014	30	10, 0	10	55, 0, 0, 0	23, 0	23	69, 0, 0, 0	25, 0	26
	2015	43	39, 0	39	96, 0, 0, 0	88, 0	88	99, 0, 0, 0	99, 0	97
	2016	44	6, 46	52	82, 21, 0, 0	9, 108	117	100, 15, 1, 1	24, 105	130
	2017	13	0, 29	29	21, 13, 2, 2	0, 64	64	23, 11, 3, 2	2, 69	71
Tech.		D	D, P	D	D, P, HP, HD	D, P	D	D, P, HP, HD	D, P	D
		Cluster 4			Cluster 5					
ω		0	0.5	1	0	0.5	1			
Year	2014	40, 0, 0, 0, 0	15, 0, 0, 0, 0	15	30, 0, 0, 0, 0	10, 0, 0	11, 0			
	2015	58, 0, 0, 0, 0	57, 0, 0, 0, 0	57	41, 0, 0, 0, 3	44, 0, 0	43, 0			
	2016	61, 3, 1, 2, 2	57, 17, 1, 1, 0	76	36, 1, 1, 2, 10	37, 16, 4	56, 1			
	2017	13, 3, 2, 2, 3	14, 24, 1, 1, 1	41	8, 1, 2, 2, 5	3, 27, 1	29, 2			
Tech.		D, P, HP, HD, E	D, P, HP, HD, E	D	D, P, HP, HD, E	D, P, E	D, E			

(c) Medium Vans

		Cluster 1			Cluster 2			Cluster 3		
ω		0	0.5	1	0	0.5	1	0	0.5	1
Year	2014	10	3, 0	3	16, 0, 0, 0	5	5	2, 0, 0, 10	3, 0, 0, 0, 1	4, 0, 0, 0
	2015	14	14, 0	13	18, 1, 0, 0	20	20	5, 0, 0, 6	10, 0, 0, 0, 3	13, 0, 0, 0
	2016	14	6, 10	17	20, 4, 0, 0	18, 9	27	7, 1, 1, 6	11, 1, 1, 1, 4	14, 1, 1, 3
	2017	4	1, 8	9	5, 2, 1, 1	6, 10	16	2, 1, 1, 3	3, 2, 1, 1, 3	6, 0, 1, 2
Tech.		D	D, P	D	D, P, HP, HD	D, P	D	D, HP, HD, E	D, P, HP, HD, E	D, HP, HD, E

4.5.2 Portfolio Effect of Clustering

In this section we want to test whether clustering decreases risk and (or) expected cost. Table 4.10 depicts the results for different capacities of vehicles for combined clusters. It can be seen that, for all vehicle types, the portion of diesel technology is the highest in comparison with other technologies, for all values of ω. For example, for ω equal to 0, the percentage of new leased diesel vehicles from 2014 to 2017, with respect to other technologies, is 94%, 96%, and 83% for small, light, and medium vans, respectively. These ratios are obtained by dividing the corresponding number of diesel vehicles by the total number of vehicles in each type of capacity.

Table 4.10: Optimal number of vehicles and technologies for combined clusters. For each cluster, and ω equal to 0, 0.5 and 1, we report the technologies leased and the number of vehicles per technology.

		Small Vans			Light Vans			Medium Vans		
ω		0	0.5	1	0	0.5	1	0	0.5	1
Year	2014	283, 4, 3, 0	82, 7, 3	92, 0	273, 3, 3, 0	82, 3, 2	87, 0	41, 1, 1, 0, 0	11, 1, 0, 0	12, 0, 0, 0
	2015	376, 17, 2, 0	362, 12, 2	321, 20	365, 8, 2, 0	356, 6, 2	318, 5	42, 4, 4, 2, 1	46, 2, 1, 1	47, 0, 0, 0
	2016	321, 29, 2, 2	71, 345, 3	428, 26	306, 19, 4, 3	102, 287, 1	417, 14	35, 5, 3, 3, 1	40, 16, 3, 2	56, 4, 1, 1
	2017	91, 7, 0, 0	4, 276, 0	233, 17	87, 4, 0, 0	5, 231, 0	226, 10	10, 1, 1, 0, 0	10, 18, 2, 2	29, 3, 1, 1
Tech.		D, P, HP, HD	D, P, HP	D, P	D, P, HP, HD	D, P, HP	D, P	D, P, HP, HD, E	D, P, HP, HD	D, P, HP, HD
Total		1137	1137	1137	1077	1077	1077	155	155	155

To consider the effect of clustering for each vehicle capacity, as seen in Figures 4.7, 4.8, and 4.9, we have obtained the values of CVaR and the expected cost per vehicle, in the case of combined clusters and different clusters. Because the number of vehicles in each cluster is different, and as we have a different expected mileage and fuel consumption for each cluster, in order to compare the CVaR values of the different clusters we obtain the CVaR per vehicle, in each cluster, for three values of ω, by subtracting the CVaR from the expected cost of each cluster, and dividing the result by the number of vehicles in the cluster. In so doing, the amount of loss (risk) for each cluster is normalised, and this value is comparable among clusters. In addition, the expected cost analysis is on a per vehicle basis. In Figures 4.7, 4.8, and 4.9, we present the results in a two dimensional setting with four quadrants with different ranges of CVaR and expected cost per vehicle. The first quadrant corresponds to the clusters with high CVaR and high expected cost per vehicle. The second quadrant includes clusters that have low CVaR and high expected cost per vehicle. The third quadrant contains clusters with low CVaR and low expected cost per vehicle. Finally, the fourth quadrant represents clusters with high CVaR and low expected cost per vehicle. Next, we analyze he common patterns observed in Figures 4.7, 4.8, and 4.9.

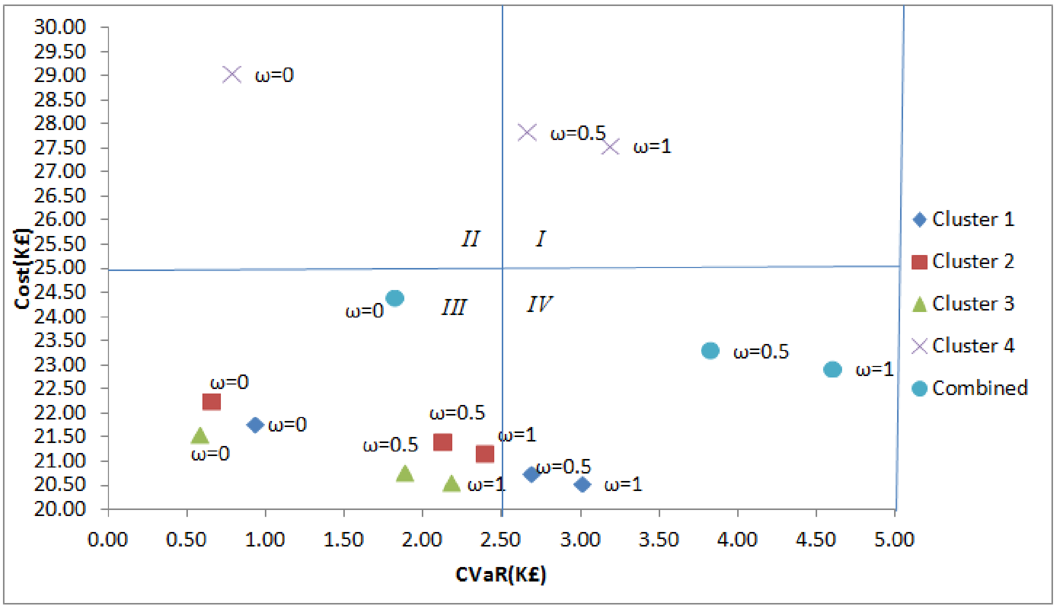

Figure 4.7: Portfolio effect on CVaR and expected cost for small vans

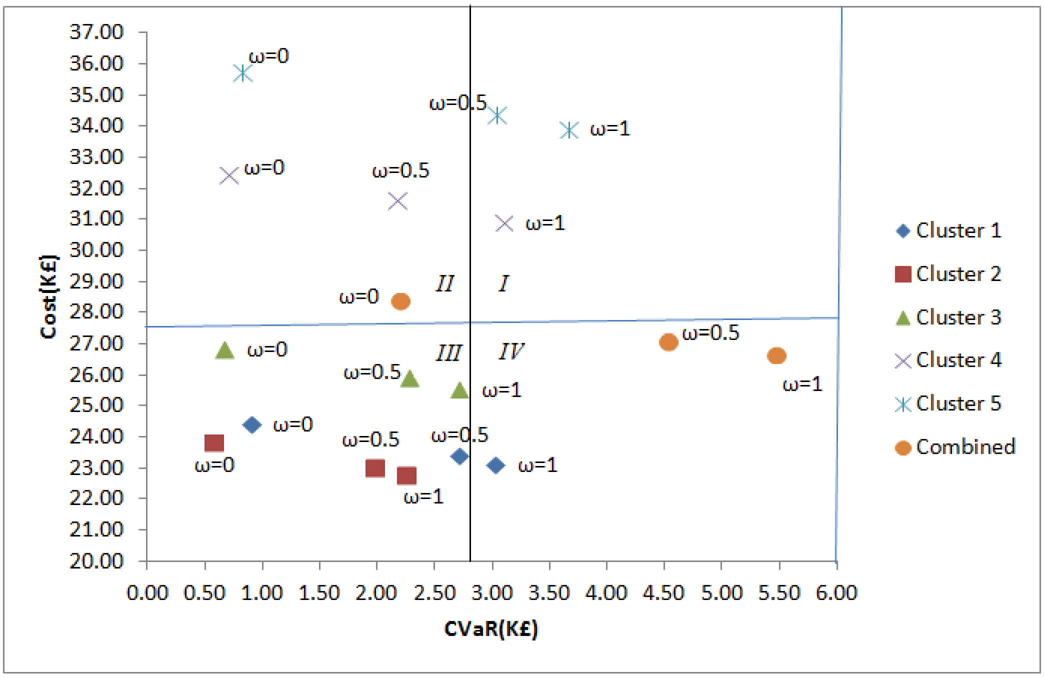

Figure 4.8: Portfolio effect on CVaR and expected cost for light Vans

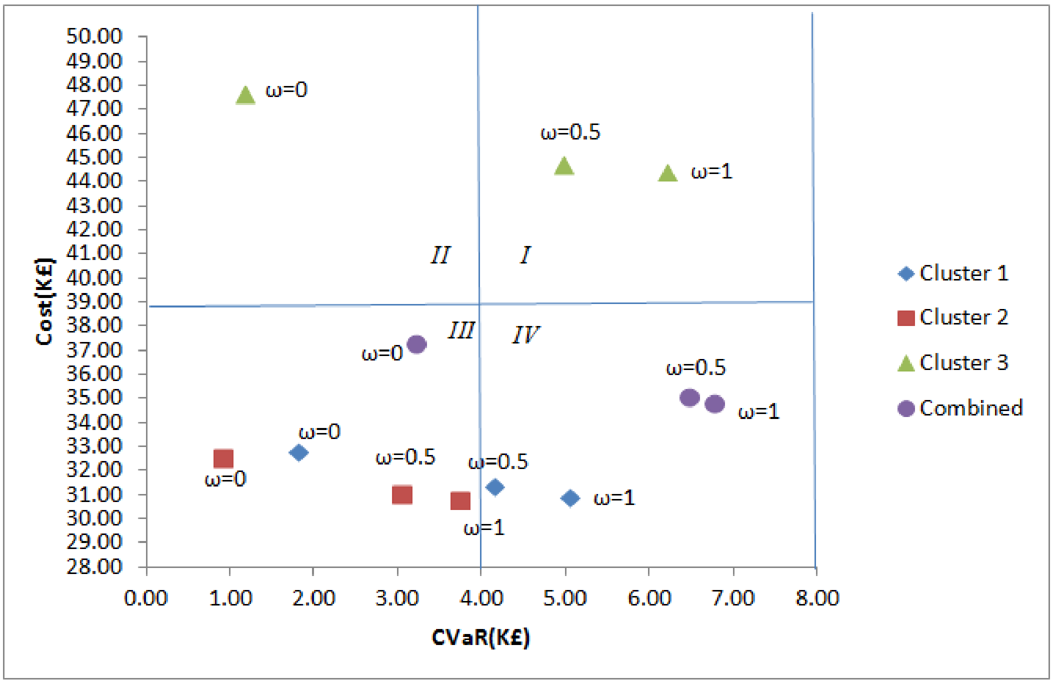

Figure 4.9: Portfolio effect on CVaR and expected cost for medium vans

For all capacities, and all values of ω, the CVaR decreases when we have clustering. For example, for the case of ω equal to 0.5, there is a reduction in CVaR per vehicle of £1.16K (30.4%), £1.94K (50.64%), £1.70K (44.44%), and £1.14K (29.73%) for clusters 1, 2, 3, and 4 in small vans, respectively. Moreover, as proved in Proposition 4.3, the amount of CVaR per vehicle for combined cluster is less than the sum of CVaR for other clusters (Subadditivity property) and is more than the average of CVaR for other clusters for each value of ω.

In addition, the expected cost per vehicle decreases by clustering, for the different vehicle capacities except for high mileage clusters when compared to combined cluster. For instance, for the case of ω equal to 0.5, there is a reduction in expected cost per vehicle of £3.72K (13.7%), £4.07K (15.2%), and £1.18K (4.33%) for clusters 1, 2, and 3 of light vans.

However, for the high mileage clusters, the expected cost per vehicle increases by clustering, for the different vehicle capacities. One reason for this increase is that adoption of EVs at high mileage clusters increases the expected cost per vehicle. Another reason is that in these clusters, we have a percentage of diesel and petrol vehicles and these vehicles are not cost efficient at high mileages. For instance, in cluster 4 of small vans, the expected cost per vehicle increases for all values of ω. This is the same in clusters 4 and 5 of light vans, and cluster 3 of medium vans. For example, for the case of ω equal to 0.5, there are increases of £4.53K (19.45%) in cluster 4 of small vans, £4.53K (16.70%) and £7.25K (26.71%) in clusters 4 and 5 of light vans, £9.66K (27.52%) in cluster 3 of medium vans, respectively.

4.6 Summary

In this Chapter, we present a general model for using different technologies in the fleet replacement decisions of companies, simultaneously taking into account the minimisation of both cost and risk. We have also obtained insightful results regarding the importance of risk in fleet management.

In terms of methodological contribution, we have developed a new formulation of a time consistent CVaR in a dynamic setting. Our approach is different from that of Shapiro (2011), in which conditional risk mapping was used to satisfy the time consistency principle. Our approach is also different from Boda and Filar (2006) in which a target-percentile approach was suggested for following the time consistency rule. Specifically, we have provided a recursive formula for the calculation of CVaR, which at each parent node takes into account the effect of the CVaR of children nodes. In addition, we have proved the time consistency and convergence properties of our CVaR formulation, and we have analyzed how clustering affects the CVaR, proving that the average CVaR of the clusters is always less or equal to the CVaR without clustering.

By considering a real case study, and performing a cluster analysis of vehicles with different capacities, based on mileage and fuel consumption as grouping variables, the major conclusions are as follows:

1. For clusters with low mileage (500 miles/month) and average mileage (1000 miles/month) with fuel efficiencies of all type of vehicles, diesel technology is the dominant choice for risk or cost minimisation purposes (ω is equal to 0 or 1). In addition, for high mileage clusters (more than 1300 miles/month), diesel technology is the dominant choice for all values of ω. However, trading diesel

technology for high mileage clusters imposes a significant amount of risk and cost to the fleet system.

2. Petrol technology is an optimal choice for risk and cost minimisation (ω is not equal to 0 or 1), in the in low and average mileage driven clusters of different type of vehicles. For example, for cluster one of small vans when ω equals to 0.5, we should lease 61% of the whole new leased vehicles with petrol vehicles. However, in low and average mileage driven clusters, when we have risk or cost minimization (ω is equal to 0 or 1), as mentioned before diesel technology is the dominant optimal choice.
3. The Hybrid-petrol and Hybrid-diesel technologies, which were also considered in the model, cannot compete with diesel technology in terms cost efficiency because of the high leasing/ownership costs. Their penetration ratio in different type of vehicles changes between 1% to 3%. If their leasing costs due to mass production are reduced to the level of diesel technology, they can be replaced for clusters with low and normal mileage profiles instead of diesel vehicles.
4. EVs generally contribute to high mileage clusters of different capacities of the vehicles. For example, for cluster 5 of light vans (which is considered for high mileage light vans) with ω equal to 0, the penetration ratio of EVs is 13%. However, by increasing the value of ω, this ratio will decrease to 4% and 2% for a ω equal to 0.5 and 1, respectively. In addition, this ratio for cluster 3 of medium vans increases to 56%, 24%, and 11% for a ω equal to 0, 0.5, and 1, respectively. Finally, for small vans EVs only contribute to cluster 4 with a penetration ratio of 2% for a ω equal to 0.
5. For all values for ω and all capacities, the risk per vehicle is reduced by clustering when compared to combined clusters. However, the expected cost per vehicle is

not reduced by clustering for clusters with high mileage profiles. This is because of the adoption of EVs at high mileage clusters which increases the expected cost per vehicle. In addition, we have a percentage of diesel and petrol vehicles, and these vehicles are not cost efficient at high mileages.

6. Finally, we found both analytically and numerically, that the CVaR per vehicle for the combined cluster is more than the average of CVaR for the separate clusters, and is less than the sum of CVaRs.

In the next Chapter, we extend our work to consider the flexibility for leasing the vehicles using different options for contracts. By doing this, vehicles can be returned during leasing period or can be swapped by other technologies by paying the price of the options and the penalties. As a result, the fleet replacement model is more comprehensive for considering the effect of technological change of EVs and evaluating the impact of different uncertainties by using real option theory and CVaR.

CHAPTER 5

FLEXIBLE LEASE CONTRACTS IN SUSTAINABLE FLEET REPLACEMENT: A REAL-OPTIONS APPROACH

Au chapitre 5, nous prolongeons le travail détaillé au chapitre 4 en prenant en considération les contrats de location souples. Dans un tel cadre, les décisions sont actualisées à chaque période avec différentes options.

Bien sûr, dans un tel contexte, nous disposons d'une grande souplesse en utilisant des contrats assortis de différentes options. Tout d'abord, nous procédons à une revue de la littérature en ayant recours à des options réelles. Puis nous appliquons notre modèle examiné au chapitre 4, avec utilisation de la CVaR et de différents contrats à option, à savoir le contrat de base (le contrat sans option), le véhicule rendu et l'échange. Dans ce chapitre, nous prenons également en considération, pour des politiques optimales de remplacement, le développement technologique des batteries, attendu à l'horizon qui est celui du plan. Enfin, nous présentons des résultats analytiques afin d'expliquer comment les contrats à option affectent la CVaR et le coût total attendu que nous appliquons dans l'analyse d'une étude de cas.

The leasing market is an important topic that should be considered for fleet replacement decisions. Leasing companies and (large) firms negotiate and bargain about a lease agreement and its conditions. Then, after negotiating with different leasing companies a firm decides which lease company wins the contract. At present a commercial lease contract usually has a standard duration of four years in the case of company we have analyzed. However, there are still many developments expected with EVs and improved models that will become available in the coming years (e.g. improvement of batteries). Therefore, firms might rather wait and obtain a better model for EVs within few years. On the other hand, for other technologies like diesel and petrol, due to governmental interventions, increasing fossil fuel prices, and other issues, they may not be economically efficient in the future. So, one could also negotiate shorter lease contracts for EVs. However, this causes a higher depreciation and, therefore, higher monthly lease costs for EVs which makes them less efficient. In contrast, for diesel and petrol cars if there is a large increase in fossil fuels prices, because of their high running costs, maybe it will be preferable to choose contacts with two years period. Another point that should be mentioned about diesel and petrol vehicles is that, because of their mature technologies, there is high bargaining power for their lease price for firms even with shorter lease contracts. Now, we want to extend the model presented in Chapter four to consider flexibility for leasing contracts, taking into account the uncertainties. For example, we can consider a lease contract which let us make our decision for returning, or to swap the vehicle with the specific technology, at the beginning of each year, during the four years planning horizon, depending on the realization of the stochastic parameters, such as CO_2 and fuel prices. In order to value different leasing option contracts, we can use real option theory, e.g., Myers (1977).

The terminology "Real Options" can first be referred to Myers (1977), who analyzed investments in real assets as options. Indeed, a real option is a permit with a changing value at different time points in order to make business decisions; for instance, an option to make a capital investment. In contrast to financial options, a real option is not tradable. For example, a firm owner cannot sell the option to extend his company to another person, only he can make this decision for his company. The term "real options" is somewhat new, whereas business companies have been making capital investment decisions for many years (e.g, Anderson, 2014).

In addition, short-term lease contracts are a characteristic of many business companies; for example, apartment leasing or service operations that involve with expensive facilities for doing special tasks. In lease contracts, lessees require services that have a short period in comparison with the life cycle of the equipment and may be repeated, possibly at various places, for many times. Lease contracts often have options influencing the lease length, especially extension and cancelation. We emphasize that the flexibility that we mentioned in the definition of a real option may take into account the acquisition of an asset or be related to the use of an asset. However, for leasing contracts, the asset produces cash when being leased or used for doing the special services; the options in the lease contract influence on this cash flow. Options on leasing contracts in different applications have been studied by Grenadier (1995) and Trigeorgis (1996).

In Section 5.1 we go through a broad literature review for real options and its different applications, specifically in environmental and sustainability development issues, and then in Section 5.2 we develop a model for considering different options for contacts. In Section 5.3 we present the analytical results. In Section 5.4 we provide a real case study and then, in Section 5.5 we develop our insights from the solved model. In Section 5.6, we extend our analysis by considering the technological change of EVs and, finally, in Section 5.7, we conclude the chapter.

5.1 Literature Review on Real Options

Real Options theory or Real Options Valuation (ROV) approach is founded on similarities between investment opportunists and financial options. A real option is a permit, but not an obligation, for doing an investment for a specific cost during or a time period. With the ROV methodology, a project is assumed an option for the generated cash flows and the optimal investment policies are just the optimal exercise rules of the option (e.g., Dyson and Oliveira, 2007).

Real options have been used widely for capital planning in many applications in different industries. Lander and Pinches (1998) categorized these applications in 16 areas: natural resources, competition and business strategy, production, real estate, R&D, public good, mergers and acquisitions, corporate governance, interest rates, inventory, labor, venture capital, advertising, legal, hysteretic effect and corporate behavior, environmental development and protection. For instance, Oliveira (2010) has studied the application of real options in strategic decision making by presenting a new formalization of strategic options as finite automata. Specifically, early real options literature was mostly found in the oil and gas upstream industry (e.g., Brennan and Schwartz, 1985; Paddock and Siegel, 1988). For example, Murphy and Oliveira (2010, 2013) have proposed the use of option contracts as instruments to manage the US Strategic Petroleum Reserve, as they signal the government commitment to act during a disruption, provide more risk-management opportunities to the refinery industry, and compensate some of the costs of maintaining the reserve.

Moreover, in the context of deregulated electricity markets, power companies face not only uncertain customer demand but also the volatility of electricity spot and forward prices (e.g., Anderson and Xu, 2006). For this reason, Sekar (2005) considered investments in three coal-fired power generation technologies by using real option assessment, considering CO_2 price as the only uncertain variable which appeared in the cash flow models of each of the three technologies. Sekar's approach used two elements simultaneously: market-based assessment for evaluation of cash flow uncertainty, and dynamic quantitative modelling of uncertainty with Monte-Carlo simulation. In addition, Laurikka (2006) developed a simulation model using real options to assess the option value of Integrated Gasification Combined Cycle (IGCC) technology within European emissions trading scheme (EU ETS). The model considered three of stochastic variables: the price of electricity, the prices of fuel and the price CO_2.). Since, our research relates to environmental and sustainability development in different industries, we focus on the literature related this application of real options.

Cortazar et al. (1998) developed an application of real options to the assessment of environmental investments. Their evaluations showed that companies, in industries with high output price fluctuation, would have more tendency to operate at low outputs levels (declining the emissions) than to invest in environmental protection projects.

Avadikyan and Llerena (2010) by taking a real options reasoning approach provide a more robust justification of companies' investment decisions on hybrid vehicles

(HVs) as a technological strategy in order to be flexible with facing market and policy uncertainties. Specifically, they introduce four types of growth options strategies on HVs which are: (1) an option to keeping the existing technological situation and providing a hedging long strategy for facing with uncertainties; (2) an option to limit HV project risks; (3) an option to diversify and (4) a platform with inside flexibility option.

Kleindorfer et al. (2012) have developed the EV adoption decision considering the affects of uncertainty in fuel, carbon, and battery prices by using a real option approach. They solve a model for optimal EV-Internal combustion vehicles (ICV) replacement decisions in a dynamic setting in the fleet system of a company in Postal sector in France. In their assumptions there is no flexibility in terms leasing contract and they have assumed a six year planning horizon for it. Moreover, they have considered four replacement policies which are (1) ICV-only policy; (2) static policy; (3) Dynamic policy; and (4) Perfect information policy.

To sum up, we understand that there is also a gap in the literature which is to view the problem of sustainable fleet replacement, taking into account the inherent flexibility that can exist for leasing contracts due to existence of uncertainties, by using real options analysis. Specifically, our approach is different with Kleindorfer et al. (2012), in terms of stochastic parameters in the model and different options for the leasing contracts. In this Chapter we extend the problem that we considered in Chapter 4 taking into account the uncertainties that exist in the real situation. These uncertainties are CO_2 prices, fuel prices, mileage driven by a vehicle and fuel consumption. We consider a lease contract which allows us to have three options for leasing the vehicles. The first choice is the base contract that has no option during four years. However, if the car is returned the penalty cost is very high. The second alternative is to lease the vehicle with option to return the vehicle by paying a small penalty. Finally, the third choice is to lease the vehicle with the swap option in which we pay a small penalty for returning the vehicle and choosing other vehicle. In order to value the aforementioned leasing contracts, we can use real option theory and then we develop our dynamic model in our previous research for considering the optimal number of cars in the leasing period.

5.2 A Multi Stage Stochastic Model with Flexible Lease Contracts

In this part we introduce a multi-stage stochastic programming model in order to obtain the optimal number of the vehicles to be leased, taking into account the constraints that exist in order to minimize a cost function which considers expected cost, and CVaR, during the planning horizon.

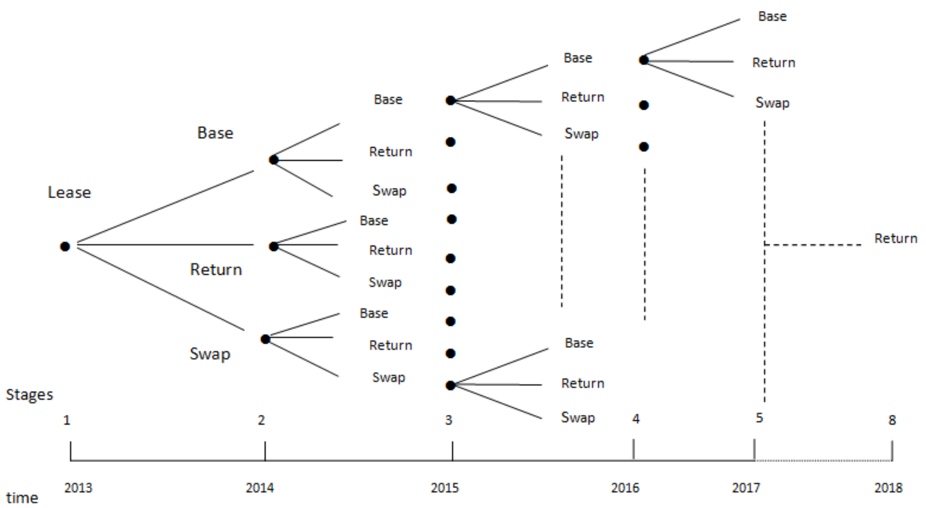

Figure 5.1: The node-based tree for a generic lease contact with options to choose the base contract, return early, and to swap.

As shown in Figure 5.1, we consider three type contracts including different options. The first choice is the base contract that has no option during four years. However, if the car is returned the penalty cost is very high. The second one is to lease the vehicle with option to return the vehicle by paying a small penalty. Finally, the third one is swap contract in which we pay a very small penalty for returning the vehicle and selecting other vehicle. In addition, in Figure 5.1, at each node, we have a vector of stochastic processes which are fuel prices, CO_2 prices, mileage driven, and fuel consumption for fossil fuel technologies, per 100 km.

Table 5.1(a). The indices, decision variables of the model.

$i \in I$={fosseil fuels, hybrids, and electric}

$a \in A$= {1, 2,.., A} index for age of the vehicles

$n \in N$= {1,2, ...,N }index for nodes in scenario tree

$t \in T$= {1, 2, .. ,T}index for time periods in year over planning horizon

$s_t \in S$= {1, 2,.., S_t } index for number of branches (states) at each stage

$c \in C$={1,2, ...,C} index for different type of contracts

$\Psi_{n,m}$: The tree structure for parent nodes n and child nodes m

$\eta_{t,n}$: The tree structure for parent nodes n and stage t

x_{niac}: The total number of vehicles with technology i, age a, contract c currently leased at node n

y^{+}_{nic}: The number of new vehicles with technology i, contract c, which company leases at node n

y^{-}_{niac}: The number of new vehicles with technology i , age a , contract c, which company returns at node n

α_n^β : Value at risk at confidence level of β at node n

Π_n : Auxiliary variable for linearization of minimum function

ϕ_n^β : Conditional value at risk at confidence level of β at node n

z_n: Auxiliary stochastic variables for loss function at node n

Table 5.1(b). Parameters of the model

W: Conversion coefficient of mileage to km

ρ : Coefficient for relation between value and price of an option

ω: Parameter for trade-off of risk and cost in the objective function

β : Confidence level for calculating CVaR and VaR

L_n: Loss function at node n

V°_{niac} : Value of the option for technology type i, age a, contract c at node n

P_{niac} : Premium of the option for technology type i, age a, contract c at node n

γ_{iac} : Penalties for returning the vehicles with technology i, age a, contract c

θ_c : The coefficient vector for penalties of different contacts c

δ : The annual learning rate for the technological development of batteries for EVs

h_n: The initial condition of the fleet system with fossil fuel technology at node n

f_{ni}: Fuel price for technology i, at node n

o_n: Fuel consumption at node n

D_n: Monthly mileage driven at node n

Q_{nia} : Expected cost per vehicle for technology i, age a, at node n

Q_n: Total expected cost function at node n

r_{ni}: Running cost per 100 km for technology i, at node n

c^p_n : The CO_2 prices at each node n

c^e: The CO_2 emissions (gr) per km for electrical technology

c^g_i : The CO_2 emissions (gr) per litre for fossil fuel and hybrid technology

l_i: The monthly lease cost for each technology i

M_e: The monthly lease cost for batteries of EVs

λ_{ni}: The total annual running cost per vechile for technology i, at node n

μ_i: The total annual fixed cost per vehicle for technology i

We consider five technologies: fossil fuels (petrol, diesel), hybrids (petrol, diesel), and EVs. In equation (5.1) we calculate the running cost for fossil fuel and hybrid vehicles, per 100 km, r_{ni}, at each node. In equation (5.1), o_n denotes the fuel consumption of fossil fuels, and hybrids, per 100 km, at each node. In addition, the running cost for EVs, per 100 km, r_{ni}, is calculated using equation (5.2). We also take into account the cost of fuel prices for each technology and CO_2 emissions, at each node, in (5.1) and (5.2), by including the parameters f_{ni} and c_n^p, respectively. Furthermore, c^g denotes the CO_2 emissions (gr/litre) for fossil fuels, and hybrids and c^e shows the CO_2 emissions for EVs per km. Finally, W is the conversion coefficient from miles to km. In Table 5.1 (a) and Table 5.1 (b) the indices and decision variables and parameters of the model are also represented.

$$r_{ni} = o_n(f_{ni} + c_n^p c_i^g / 10^6) \qquad \forall n \in N, i = \text{fossil fuels, hybrids} \qquad (5.1)$$

$$r_{ni} = \frac{f_{ni}}{W} + 100(\mathrm{c}^e / 10^6) c_n^p \qquad \forall n \in N, i = \text{electric} \qquad (5.2)$$

As a result, based on equations (5.1) and (5.2), we calculate the total annual running cost per vehicle at each node, λ_{ni}, using equation (5.3). In equation (5.3), D_n represents mileage driven at node n.

$$\lambda_{ni} = 12 W r_{ni} / 100 D_n \qquad \forall n \in N, i \qquad (5.3)$$

The total investment (fixed) cost per vehicle is represented by (5.4) for fossil fuels and hybrid technologies and by (5.5) for EVs. As we take into account the leasing contracts for providing different types of vehicles in the fleet system, we use the monthly lease cost which is represented by l_i, to obtain the fixed cost at each node. Moreover, for EVs we have extra investment cost which is monthly lease cost for batteries which is presented in equation (5.5) by M_e.

$$\mu_i = 12 l_i \qquad i = \text{fossil fuels, hybrid} \tag{5.4}$$

$$\mu_i = 12(l_i + M_e) \qquad i = \text{electric} \tag{5.5}$$

The penalty, γ_{iac}, for returning the vehicles, is represented in equation (5.6).

$$\gamma_{iac} = \theta_c (A - a)\mu_i \qquad a<A \tag{5.6}$$

In equation (5.6), θ_c represents the coefficient vector of penalties for different type of contracts and A is the maximum age of vehicle during leasing period. Moreover, equations (5.7)-(5.9) show the value of different options.

$$V^0_{nia1} = 0 \tag{5.7}$$

$$V^0_{nia2} = \max(0, Q_{nia} - \gamma_{ia2}) \tag{5.8}$$

$$V^0_{nia3} = \max(0, Q_{nia} - \min_{i \neq j}(Q_{nij}) - \gamma_{nia3}) \tag{5.9}$$

Equation (5.7) represents the value of contract without any option i.e., base contract. Equations (5.8) and (5.9) represent the value of the contracts including return and swap options, respectively. In addition, in equations (5.7)-(5.9), Q_{nia} is the expected cost per vehicle for each technology, at age *a*, which is calculated by equation (5.10).

$$Q_{nia} = \mu_i + \lambda_{ni} + \frac{1}{S_t} \sum_{\psi(n,m)} Q_{mi(a+1)} \tag{5.10}$$

Finally the premium of each option, P_{niac}, is obtained by equation (5.11),

$$P_{niac} = \rho V^0_{niac} \tag{5.11}$$

In which ρ is a parameter between 0 and 1.

Our objective is to minimize the weighted average of CVaR and cost at the root node. Each firm aims to solve the mixed integer multi-stage stochastic programming (MIP) model in equations (5.12)-(5.25):

$$\underset{x_{niac},y^{+}_{nic},y^{-}_{niac},\alpha_n^{\beta}}{Min}\ \omega Q_1+(1-\omega)\phi_1^{\beta} \tag{5.12}$$

s.t.

$$\sum_c x_{1i1c}=h_n \quad \forall n\text{ , } i\text{=fossil fuel} \tag{5.13}$$

$$y^{+}{}_{nic}=x_{ni1c} \quad \forall n\in N, i\in I \tag{5.14}$$

$$y^{+}{}_{nic}\text{ and } y^{-}{}_{niac}=0 \qquad \forall n\in\Omega_{t,n}\ \text{ if } t\geq T\text{-3, } a\text{=1} \tag{5.15}$$

$$x_{miac}=y^{+}_{mic}+(x_{ni(a-1)c}-y^{-}_{miac}) \quad \forall a\in A,\forall(n,m)\in\Psi_{n,nn} \tag{5.16}$$

$$\sum_i\sum_a\sum_c x_{niac}\geq h_n \qquad \forall n\in\Omega_{t,n},\ t\leq T-3 \tag{5.17}$$

$$\begin{aligned}L_n=&\sum_i\sum_a\sum_c(\lambda_{ni}+\mu_i+P_{niac})x_{niac}/10^6+\sum_i\sum_{a=2}^{A}(\gamma_{ia1}(y^{-}_{nia1}+y^{-}_{nia3}))\\&+\sum_i\sum_{a=2}^{A}\gamma_{ia2}(y^{-}_{nia2})\text{ -}\sum_i\sum_{a=2}^{A}(\gamma_{ia1}-\gamma_{ia3})\Pi_n \qquad \forall n\in\mathrm{N}\end{aligned} \tag{5.18}$$

$$\Pi_n\leq\sum_i\sum_{a=2}^{A}y^{-}_{nia3} \quad \forall n\in N \tag{5.19}$$

$$\Pi_n\leq\sum_i\sum_c y^{+}_{nic} \quad \forall n\in N \tag{5.20}$$

$$Q_n = L_n + \frac{1}{S_t} \sum_{\Psi(n,m)} (Q_m) \qquad \forall (n,m) \in \Psi_{n,m} \tag{5.21}$$

$$z_m \geq L_m - \alpha_n^{\beta} \qquad \forall (n,m) \in \Psi_{n,m} \tag{5.22}$$

$$\phi_n^{\beta} = \alpha_n^{\beta} + \frac{1}{S_t(1-\beta)} \sum_{\Psi(n,m)} (z_m) + \frac{1}{S_t} \sum_{\Psi(n,m)} (\phi_m^{\beta}) \qquad \forall (n,m) \in \Psi_{n,m} \tag{5.23}$$

$$\alpha_n^{\beta} = 0 \qquad \forall \mathrm{n} \in \Omega_{t,n} \tag{5.24}$$

$$x_{niac}, y_{nic}^{+}, y_{naic}^{-}, \Pi_n \in Z^{+},\ \alpha_n^{\beta},\ z_n \in \mathrm{R}^{+} \tag{5.25}$$

The objective function (5.12) minimizes the weighted average of expected cost, Q_1, and CVaR, ϕ_1^{β}, at the root node. Equation (5.13) shows the initial condition of the fleet system at each node, h_n, which is composed of new leased fossil fuel vehicles, should be equal to total number of the vehicles with different type of contracts, $\sum_c x_{1i1c}$, at the root node. In equation (5.14) we determine the number of new leased vehicles with specific contract, at each node, y^{+}_{nic}, required to replace the segment of new vehicles of the total vehicles in the fleet system, x_{ni1c}, due to retirement of the older vehicles at the corresponding node. In addition, equation (5.15) shows that planning horizon for decision variables, y^{+}_{nic} and y_{niac}^{-} is four years and after that there will be no new leased vehicles, and returned vehicles in the fleet system. In addition, the vehicles that are at age one (a=1) cannot be returned. Equation (5.16) shows that the total number of the vehicles, at each child node, x_{miac}, is equal to number of new leased vehicles, y^{+}_{mic}, plus the number of vehicles which are left , $x_{ni(a-1)c} - y^{-}_{miac}$, after returning vehicles, y^{-}_{miac}, with age more than one. Moreover, equation (5.17) represents that the total number of vehicles for all technologies, contracts, and ages, $\sum_i \sum_a \sum_c x_{niac}$, at each node , should be greater than or equal to the number of vehicles which are needed, h_n, at the corresponding node during first four years. Equation (5.18), shows the total loss function (total cost), L_n, at each node. The first term in equation (5.18), $\sum_i \sum_a \sum_c (\lambda_{ni} + \mu_i + P_{niac}) x_{niac} / 10^6$, represents the sum of running cost, λ_{ni}, fixed cost, μ_i, premium of the options, P_{niac}, at the corresponding node. The second term, $\gamma_1 \sum_i \sum_{a=2}^{A} (y^{-}_{nia1} + y^{-}_{nia3})$, shows the penalty cost for returning the vehicles for base and swap contracts, and the third term, $\gamma_2 \sum_i \sum_{a=2}^{A} (y^{-}_{nia2})$, represents the penalty cost for contracts with option of return. Finally, the fourth term, $(\gamma_1 - \gamma_3)\Pi_n$, shows the amount of money that is given back when the swapping option is selected. Equations (5.19-5.20) represent constraints for linearization of minimum function, $\Pi_n = \min(\sum_i \sum_c y^{+}_{nic}, \sum_i \sum_{a=2}^{A} y^{-}_{nia3})$, used in equation (5.18). Equation (5.21) presents the recursive formula for calculating the total expected cost function, at each node, Q_n, which is equal to the loss function, L_n, at the corresponding node plus the average of cost functions, $\frac{1}{S_t} \sum_{\Psi(n,m)} (Q_m)$, in successor nodes. In order to take into account the time consistency issue of CVaR (Shapiro, 2011), we have used equations (5.22) and (5.23) which have been proved before in Chapter 4. In addition, equation (5.24) shows that the Value at Risk (VaR), α_n^β , at the final stage should be zero due to the fact that at the final stage there is no uncertainty and all values of stochastic processes are realized. Finally, (5.25) is the constraint for the integer values of, x_{niac}, y^+_{nic}, y^-_{niac}, Π_n, and non-negative, α_n^β and z_n, decision variables.

5.3 Analyzing the Main Properties of the Model

In this section we want to consider the impact of using option contracts on the expected cost of the fleet. So, we want to test if by using option contracts, we can reduce the expected cost of the fleet system. This is the conventional goal of risk neutral fleet managers. In order to provide a formal proposition for it, first in Proposition 5.1 we provide a prerequisite proposition and then after that we proceed for the main proposition related to the effect of using option contracts on the expected cost of the fleet in terms cost efficiency.

Let ΔL_{nic} stand for the change in the loss function per vehicle, type i, for contract c, at node n, compared to the case without using any contract c, ΔQ_{niac} represent the change of expected cost per vehicle for contract c, at age a, at node n, compared to the case without using any contract c, $E(\Delta L_{mic})$ be the expected change of loss function per vehicle, type i, at child m, at age one, compared to the case without using any contract c, and $EE(\Delta L_{m^2c})$ represent the expected of expected change of loss function per vehicle, type i, at child node of child node m, at age 2, compared to the case without using any contract c, and $EE....E(\Delta L_{m^Aic})$ represent the expected of expected change of loss function per vehicle, type *i*, at child node of child node *m*, at age *A*, compared to the case without using any contract *c*

Proposition 5.1: *If* $0 < \gamma_{ia3} < \gamma_{ia2} < \gamma_{ia1}$ *then the change of the expected cost at parent node n,* ΔQ_{niac} *, for contract c , at age a, compared to the case without using any contract c is :*

$$\Delta Q_{niac} = \Delta L_{nic} + E(\Delta L_{mic}) + EE(\Delta L_{m^2ic}) + ...EE....E(\Delta L_{m^{A-1}ic}) + EE....E(\Delta L_{m^Aic})$$

Proof: In order to calculate the values of ΔQ_{niac}, based on equation (5.18), we can obtain (5.26). Because (5.26) is a recursive equation, in which we can replace the value of $E(\Delta Q_{miac})$ by using equation (5.18) for child node *m*, and if we continue it until the maximum age of the contract, *A*, we can derive (5.27).

$$\Delta Q_{niac} = \Delta L_{nic} + \frac{1}{S_t}\sum_{\Psi(n,m)}(\Delta Q_{miac}) = \Delta L_{nic} + E(\Delta Q_{miac}) \tag{5.26}$$

$$\Delta Q_{niac} = \Delta L_{nic} + E(\Delta L_{mic}) + EE(\Delta L_{m^2c}) + ... + EE...E(\Delta L_{m^Aic}) \ \blacksquare \tag{5.27}$$

Now, we proceed for the main proposition related to the effect of using option contracts on the expected cost of the fleet in terms cost efficiency.

Let V^b_{niac} denote the ex-ante value of the option contract *c*, for technology *i,* at age *a*, and at node *n* and $\bar{\lambda}_{(A-a)ic}$ denote the average of running cost for the remaining periods at age *a*, for technology *i,* and option contract *c*.

Proposition 5.2: *If* $0 < \gamma_{ia3} < \gamma_{ia2} < \gamma_{ia1}$ *then*

$$V^b_{niac} = (\Delta Q_{niac} - \gamma_{niac})^+ = (A-a)[\bar{\lambda}_{(A-a)ic} + \Delta\mu_i - \theta_c \mu_i)]^+ \quad \forall c,\ a \geq 2$$

Proof: In order to calculate the values of V^b_{niac} for each contract, we use equation (5.28) in which we have benefited from the result of Proposition 5.1 in equation (5.27). In equation (5.28), ΔL_{nic}, ΔL_{mic},..., and $\Delta L_{m^A ic}$ in all terms have two components. The first one is $\Delta\mu_{ic}$ which denotes the change in the annual fixed cost per vehicle type *i*, for contract *c* compared to the case without using any contract *c*, and the second component is $\Delta\lambda_{nic}$, $\Delta\lambda_{mic}$, $\Delta\lambda_{m^2 ic}$,..., and $\Delta\lambda_{m^A ic}$, which represent the change of the annual running cost per vehicle, at node parent *n*, child node *m*, child node of child node *m*, and so on, for contract *c* compared to the case without using any contract *c*. Now, based on equation (5.28), we can derive (5.29) in which the first term, $\Delta\mu_{ic} + \Delta\lambda_{nic}$, equal to zero because we do not return any vehicle at age one and the second term denotes the expected change of cost per vehicle which is incurred by having contract *c*, age 2, and so on.

Moreover, if we let $E(\Delta\lambda_{nic}) = \bar{\lambda}_{(A-a)ic}$, we can derive (5.30) from which we obtain, V^b_{niac}, (5.31).

$$\begin{aligned}\Delta Q_{niac} &= \Delta L_{nic} + E(\Delta L_{mic}) + EE(\Delta L_{m^2 c}) + ...EE....E(\Delta L_{m^{A-1} ic}) + EE...E(\Delta L_{m^A ic}) \\ &= \Delta\mu_{ic} + \Delta\lambda_{nic} + E(\Delta\mu_{ic} + \Delta\lambda_{mic}) + EE(\Delta\mu_{ic} + \Delta\lambda_{m^2 ic}) + EE....E(\Delta\mu_{ic} + \Delta\lambda_{m^A ic})\end{aligned} \quad (5.28)$$

$$\Delta Q_{niac} = E(\Delta\mu_{ic} + \Delta\lambda_{mic}) + EE(\Delta\mu_{ic} + \Delta\lambda_{m^2 ic}) + EE....E(\Delta\mu_{ic} + \Delta\lambda_{m^A ic}), \forall a \geq 2 \quad (5.29)$$

$$\Delta Q_{niac} = (A-a)[\bar{\lambda}_{(A-a)ic} + \Delta\mu_{ic}], \quad \forall a \geq 2 \quad (5.30)$$

$$V^{b}_{niac} = (\Delta Q_{niac} - \gamma_{iac})^{+} = (A-a)[\bar{\lambda}_{(A-a)ic} + \Delta\mu_{ic} - \theta_c \mu_i)]^{+}, \ \forall a \geq 2 \qquad \blacksquare \qquad (5.31)$$

Equation (5.31) explains that if we return a vehicle with contract c , at age a, at node n, the amount which is saved , compared to the case without using any contract c , is equal to change of expected cost per vehicle of type i, contract c, at parent node n, and at age a, minus the penalty, γ_{iac}, which is incurred if the vehicle is returned by contract c. The penalty can be obtained by equation (5.6) for each contract. The amount which is saved is equal to the ex-ante value of the option contract c, i.e., V^{b}_{niac} .

Now, we can conclude that the total cost of the fleet system decreases by using option contact c, compared to the case without using any contract c , if the sum of the ex-ante values of V^{b}_{niac} for all the vehicles for any contract c, has a positive value.

Next, we obtain the condition under which one of the option contracts is chosen instead of the other one. The general conclusion is that by selecting the appropriate values of the parameters, we have the flexibility for choosing different option contracts. This is a very important issue which gives flexibility for managers to choose the contract which is the matched with their fleet management system condition.

Let V^{g}_{niac} denotes for the ex-post value of option contract c, technology i, at age a, at node n, and τ_c represents the probability that contract c is used. Then for ex-post value of option contracts, we use (5.32). In equation (5.32), P_{niac} is the premium of option contract c, which can be obtained by equation (5.11).

$$V_{niac}^{g} = (V_{niac}^{b} - P_{niac})^{+} \tau_c \quad \forall c \tag{5.32}$$

Proposition 5.3: *If* $0 < \gamma_{ia3} < \gamma_{ia2} < \gamma_{ia1}$, $1 \le k, j \le c$ *, then the contract with ex-post value of* V_{niaj}^{g} *is used instead of the contract with ex-post value of* V_{niak}^{g} *when*

$$\frac{[\bar{\lambda}_{(A-a)ij} + \Delta\mu_{ij} - \theta_j \mu_i - P_{niaj}]^{+}}{[\bar{\lambda}_{(A-a)ik} + \Delta\mu_{ik} - \theta_k \mu_i)]^{+}} > \frac{\tau_k}{\tau_j}$$

Proof: The general condition in which the contract with the option j is used instead of contract with option k can be written in (5.33). Equation (5.33) explicitly represents that if contract with the option j is used instead of the contract with option k, then its ex-post value, should be higher. Then, by using equations (5.31) and (5.33), we can derive (5.34).

$$V_{niaj}^{g} > V_{niak}^{g} \Rightarrow (V_{niak}^{b} - P_{niak})^{+} \tau_k > (V_{niaj}^{b} - P_{niaj})^{+} \tau_j \tag{5.33}$$

$$\frac{[\bar{\lambda}_{(A-a)ij} + \Delta\mu_{ij} - \theta_j \mu_i - P_{niaj}]^{+}}{[\bar{\lambda}_{(A-a)ik} + \Delta\mu_{ik} - \theta_k \mu_i - P_{niak})]^{+}} > \frac{\tau_k}{\tau_j} \quad \blacksquare \tag{5.34}$$

Now another question is that: what is the effect of using contracts on the risk (CVaR) of the fleet management system. The answer to this question addresses the concern for risk averse fleet managers. Moreover, from methodological perspective it is one our contributions in this Chapter which considers the interaction of CVaR and using option contracts on the fleet replacing decisions. The next proposition proves that the value of CVaR decreases by using option contracts.

Proposition 5.4: *Let* ϕ_n^{β} *and* $\phi_{n(0)}^{\beta}$ *represent the value of CVaR with and without contracts, respectively. Then we have,* $\phi_n^{\beta} - \phi_{n(0)}^{\beta} \le 0 \quad \forall\ n \in N, c \in C.$

Proof: We proceed by using proof by contradiction. So, let's assume that (5.35) is true.

$$\phi_n^{\beta} - \phi_{n(0)}^{\beta} \ge 0, \ \ \forall\ n \in N, c \in C. \tag{5.35}$$

Then the optimal choice is to not use contracts and we have a lower CVaR without options, $\phi_{n(0)}^{\beta}$. On the other hand, if we have option to use the contracts and $\phi_n^{\beta} < \phi_{n(0)}^{\beta}$ then we have a lower CVaR, when we exercise the options, and this will equal, ϕ_n^{β}. Therefore, by having the option to exercise contracts we get $\phi_n^{\beta} \le \phi_{n(0)}^{\beta}$.

■

So far we have considered the effect of using the option contracts on the expected cost and on the CVaR. Now we want to answer the question what is the change of value per year for the swap option of leasing EVs instead of fossil vehicles, if we consider the technological change of batteries of EVs which is expected to be happened in the coming years. This is also an interesting issue which can be a good justification for considering EVs in the fleet replacement decisions for managers by using swap option. Before answering this question, we provide a proposition in which we can obtain the time that takes EVs to be more efficient than other technologies and then we present another proposition to study the idea that we have mentioned.

Now, we are ready to provide a proposition for calculating the time during our planning horizon in which EVs are more cost efficient than other technologies. Let

μ_{0e} stand for the annual fixed cost of the EVs, at time zero, μ_{te} denote the annual fixed cost of EVs in year *t*, and $g(\mu_{te}^{*}, t)$ represent the density function *of the First Passage Time* (*FTP*) for the stochastic process μ_{te}. The first passage time is the expected time when μ_{te} crosses a threshold (Withmore, 1986).

Proposition 5.5: *Let i stand for fossil fuel or hybrid vehicles. The expected number of years for EVs to be more cost efficient than technology i is:*

$$\int_0^\infty tg\left[12(l_i + WD_t r_{ti}/100)\text{-}WD_t r_{te}/100,\ t\right]dt$$

Proof: Because in the tree structure, in each stage *t*, we can map the set of nodes *n,* using $\eta_{t,n}$, we can use *t* and *n* and in (5.36), interchangeably. Now, in order to calculate the first passage time, we should equal the total annual cost of EVs with other technologies, i.e.,

$$\mu_{te}^{*} + \lambda_{te} = \mu_{ti} + \lambda_{ti} \quad \forall t, n \in \eta_{t,n},\ i = \text{fossil fuels, hybrids} \tag{5.36}$$

Then by using equations (5.3), (5.4), and (5.36), we drive,

$$\mu_{te}^{*} = \mu_{ti} + \lambda_{ti} - \lambda_{te} = 12(l_i + WD_t r_{ti}/100)\text{-}WD_t r_{te}/100 \quad \forall t, n \in \eta_{t,n} \tag{5.37}$$

All the parameters in (5.37) are defined in Tables 5.1(a) and 5.1(b). Then, the expected first passage time equals:

$$\int_0^\infty tg(\mu_{te}^{*}, t)dt = \int_0^\infty tg\left[12(l_i + WD_t r_{ti}/100)\text{-}WD_t r_{te}/100,\ t\right]dt \quad \blacksquare \tag{5.38}$$

Next we want to analyze the change of the value of option to swap a fossil fuel or hybrids for leasing EVs per year, if we consider the technological change of batteries of EVs. The reason for doing this is that we can swap a fossil fuel vehicle for EVs and we can reduce the expected cost. As we mentioned before this is also an interesting issue which can be a good justification for considering EVs in the fleet replacement decisions for mangers by using the swap option.

Proposition 5.6: *Let $\Delta\mu_{tie}$ denote the difference of change of annual fixed cost of EVs and technology i, in year t, and T represents the length of the planning horizon. The value of the option to swap a fossil fuel or hybrid vehicle for EVs increases, per year, by* $\Delta V_{tia3} = T\Delta\mu_{tie}$.

Proof: The value of option at time zero to swap a fossil fuel or hybrids for EVs by using equations (5.9) and (5.10) is:

$$\begin{aligned} V_{nia3}^{t_0} &= (Q_{nia} - \min_{i \neq j}(Q_{nij}) - \gamma_{nia3})^+ \\ &= (\mu_i + \lambda_{ni} + \frac{1}{S_t}\sum_{\psi(n,m)} Q_{mi(a+1)} - \mu_e - \lambda_{ne} - \frac{1}{S_t}\sum_{\psi(n,m)} Q_{me(a+1)} - \gamma_{nia3})^+ \end{aligned} \tag{5.39}$$

Because in the tree structure, in each stage *t*, we can map the set of nodes *n,* using $\eta_{t,n}$, we can use *t* and *n* and in (5.40), interchangeably. In order to calculate the change in the value of swap option at time *t*, because the only change during the planning horizon, in (5.39), is the fixed cost of EVs, we can write:

$$\Delta V_{tia3} = V_{nia3}^{t_t} - V_{nia3}^{t_0} = \Delta\mu_{tie} + \frac{1}{S_t}\sum_{t=2}^{T}\Delta\mu_{tie} = \Delta\mu_{tie} + \frac{1}{S_t}(T-1)\Delta\mu_{tie}S_t = T\Delta\mu_{tie} \quad \blacksquare \tag{5.40}$$

5.4 A Case Study on Sustainable Fleet Replacement with Flexible Leasing Contracts

At each stage (year), we want to have the minimum number of vehicles in the fleet system, taking into account that the contract of the vehicles will be retired by the end of the fourth year. Moreover, we assume an initial condition of the 2013 fleet system consisting of 2369 diesel vehicles with different capacities (small, light, medium), which all of them are at age one, i.e., new leased vehicles. Because the light vans characteristics are between small and medium vans, we selected them for considering the optimal policies. Moreover, for value of ω, we assumed ω equals to 0.5 in which the weights for expected cost and CVaR in the objective function are equal. All the values of parameters needed for the model are obtained from Tables 4.2, 4.3, 4.4, and 4.6 in Chapter 4.

There are options for returning or swapping the vehicles that are at age two, three, and four. Indeed, the vehicles that are at age one or new leased vehicles cannot be returned or swapped.

Given these constraints, we want to minimise the weighted average of total expected costs and CVaR during the planning horizon. Our goal is to determine the optimal policy from 2013 to 2016. Note that in order to calculate this policy we need to continue the calculation for the stochastic variables from 2017 to the end of 2020 (until the end of life of the vehicles leased in the period of analysis).

5.5 Result of Optimal Policies for Technologies in Light Vans

We considered three values of ρ equal to 0, 0.1, and 0.25 for relation between value and premium of an option. Moreover, the coefficient vector for penalties of different contacts, θ_c, is equal to 1, 0.5, and 0.1 for contracts with no option (c = 1), return option (c = 2), and swap option (c = 3), respectively. We denote different technologies by D, P, H-P, H-D, and E for Diesel, Petrol, Hybrid-Petrol, Hybrid-Diesel, and Electric, respectively. Finally, the results are converged by using 10165 scenarios. The results are shown in Tables 5.2, 5.3, and 5.4.

As seen from Table 5.2, when ρ equals zero, i.e., the options are free, we have different combination of contacts for leasing of vehicles. In addition, diesel technology has the biggest portion of leased vehicles in different contracts. Moreover, as expected, because the swap contract has the lowest penalty, we have a lot of swapped vehicles. For example, in 2014 we have 932 diesel vehicles at age 2, which are swapped by other contacts with the same technology. However, we do not have any returned vehicles with base contract; because it has the highest penalty.

Now we change the value of ρ to 0.1, i.e., we should pay for return and swap contracts, 10% of their value which is calculated in equations 5.8-5.9. The results are represented in Table 5.3. As seen, the vehicles are leased with base contract and contract with swap option. In addition, we have returned vehicles with base contract and contract with swap option. However, there are no returned vehicles with option to return due to the higher penalty of them in comparison with swap contracts. The interesting observation which is retuned vehicles with base contract, shows the fact that there is a trade-off between penalty and price of an option. Finally, when ρ is 0.25 or more, as represented in Table 5.4, we have just lease and retuned vehicles with base contract. In addition, there is no swap of the vehicles. We have provided a formal proof for these results in Propostion 5.3.

Table 5.2: Optimal policy for lights vans, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1), and ρ=0

			Number of Vehicles				
			Leased			Returned	Swapped
Year	Tech.	age	C1	C2	C3	C2	
2013	D	1		34	1043		
2014	D	1	210	212	551		
		2				6	932
2015	P	1	1	1			
	D	1	38	39	31		
		2				4	7
		3				11	90
2016	P	1	1				
	D	1	31				
		2				2	2
		3				4	2
		4				8	12

Table 5.3: Optimal policy for lights vans, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1), and ρ=0.1

			Number of Vehicles			
			Leased		Returned	Swapped
Year	Tech.	age	C1	C3	C1	
2013	D	1	84	993		
2014	D	1	1028	3		
		2			7	990
2015	P	1	1			
	D	1	20			
		2			3	1
		3			17	3
2016	P	1	1			
		3			2	
		4			18	
	D	1	20			

Table 5.4: Optimal policy for lights vans, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1), and ρ=0.25

			Number of Vehicles	
			Leased	Returned
Year	Tech.	age	C1	C1
2013	D	1	1077	
2014	D	1	41	
		2		7
2015	P	1	5	
	D	1	20	
		3		29
2016	P	1	3	
	D	1	50	
		4		54

Moreover, as seen from Figure 5.2, the values CVaR and expected cost, when ρ equals to zero are less than the case when ρ is not equals to zero. The reason is that using contracts with options decreases the whole CVaR and expected cost. In other words, when more contracts are used in the fleet system, the total cost and CVaR are minimized. Indeed, the lower the price of the options, the lower the CVaR and expected cost. For example, when ρ decreases from 0.25 to 0, we have £3.56M (12%) decrease in CVaR and £2.99M (11%) in expected cost. We have proved these results in Propositions 5.2 and 5.4 regarding the effect of option contracts on the expected cost and CVaR, respectively.

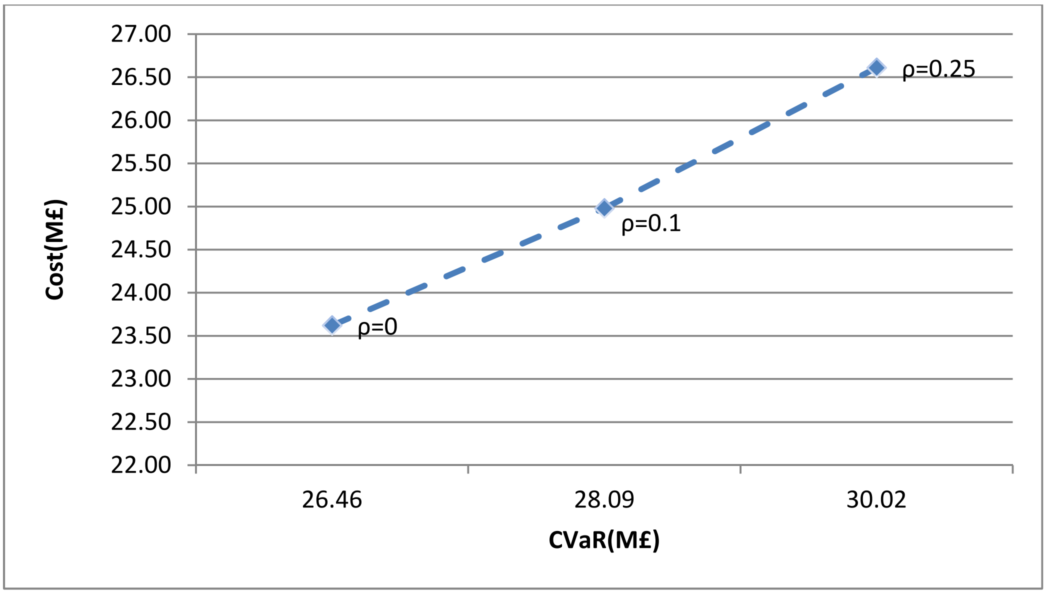

Figure 5.2 The values of CVaR and Expected cost for Lights Vans, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1).

5.6 Modeling the Effect of Technology Development

In this section we want to consider the progress that is expected for the next decade for technology of batteries and EVs market and consequently its affect on the suggested portfolio system of the company.

In the report provided by Book et al. (2009), they have predicted that in China, Japan, North America, and Western Europe, 1.5 million EVs will be sold in 2020, reflecting some 2.7 percent of the total automotive market in these regions. In terms of market segments, EVs are most likely to be introduced in the city car segment, where they will take the form of small city cars used mainly for commuting within the city. They predict that 18 percent of city cars across the four regions will be EVs in 2020 under the steady-pace scenario.

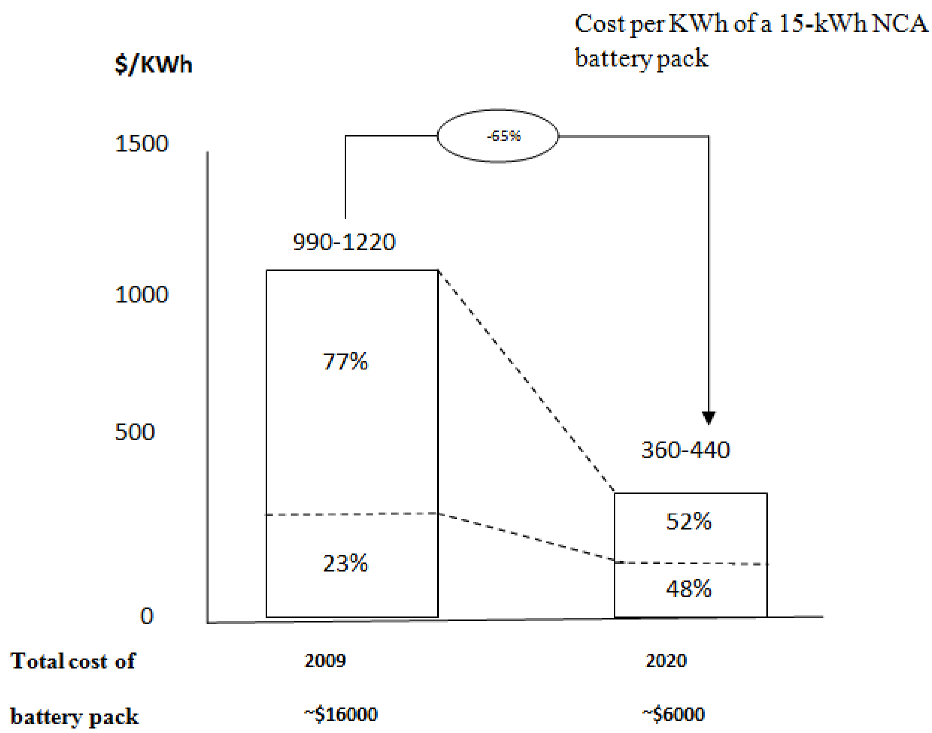

Figure 5.3: BCG outlook for Battery costs from 2009 to 2020, Dinger et al., 2010

Dinger et al. (2010), predict that battery costs will decline steeply as production volumes increase. Individual parts will become less expensive thanks to experience and scale effects. Equipment costs will also drop, lowering depreciation. However, 25 percent of current battery costs (primarily the costs of raw materials and standard, commoditized parts are likely to remain relatively independent of production and to change only modestly over time. Their analysis suggests that from 2009 to 2020, the price of NCA (Nickel Cobalt Aluminium) batteries which are mostly used in EVs will decrease 60 to 65 percent (Figure 5.3).

Dinger et al. (2010) mention that the current cost of an automotive lithium-ion battery pack, as sold between $1000 and $1200 per KWh. They further predict that this price tag will decline to between $250 and $500 per KWh at scaled production.

As a result for 15Kw battery (Figure 5.3), the price is expected to drop from \$16000 to \$6000.

As we mentioned in Proposition 5.5, we can model the effect of technology development of EVs by a *FTP* process. So, we can use Ornstein–Unhlenbeck process (e.g., Kleindorfer et al., 2012) as *FTP* process to generate the scenarios at each node for monthly rents for EVs with appropriate parameters. We have used this stochastic process because; in the long run, the battery costs and monthly rents for EVs will be declined to a steady state case like other technologies. The Ornstein-Unhlenbeck process equation is represented by (5.48).

$$\mu_{ne} = \bar{\mu}_e(1-\exp(-\delta)) + \mu_{me}\exp(\text{-}\delta) + \sigma\sqrt{\frac{1-e^{-2\delta}}{2\delta}}.z \qquad (5.41)$$

In (5.48) μ_{ne} is the total annual fixed (investment) cost of EVs at each parent node n, μ_{me} is the total annual fixed cost of EVs per vehicle at root node, and $\bar{\mu}_e$ is the average total annual fixed cost of EVs per vehicle in the long run. According to Dinger et al. (2010), we assume to be 40% of the current fixed cost of EVs. In addition, δ is the annual learning rate, z is the quantile for standard normal distribution and, finally, σ is the standard deviation of μ_{ne}.

The results are represented in Tables 5.5, 5.6, and 5.7. The learning rate, δ, is equal to 1.2 (Dinger et al., 2010). For other parameters, we have taken into account the previous assumptions in Section 5. As seen from Tables 5.5, 5.6, and 5.7, the dominant technology for leasing is EVs. In some cases we also choose Diesel. But there are no optimal choices for other technologies for leasing. Next, we consider each case in more detail for corresponding optimal policies regarding the contracts.

As seen from Table 5.5, when ρ equals zero, i.e., the options are free, we have different combination of contacts for leasing of vehicles. Moreover, as expected, because the swap contract has the lowest penalty, we have a lot of swapped vehicles. For example, in 2014 we have 808 EVs at age 2, which are swapped. However, we do not have any returned vehicles like in Section 5.6, with base contract; because it has the highest penalty.

Next, we change the value of ρ to 0.1, i.e., we should pay for return and swap contracts, 10% of their value which is calculated in equations 5.8-5.9. The results are represented in Table 5.6. As seen, the vehicles are leased with base contract, and contract with swap option, except in 2013 in which 32 diesel vehicles are leased with contract with return option. In addition, we have returned vehicles with base contract, and contract with swap option. However, there are no returned vehicles with option to return due to the higher penalty in comparison with swap contracts. The interesting observation which is retuned vehicles with base contract, shows the fact that there is a trade-off between penalty and price of an option. Finally, when ρ is 0.25 or more, as represented in Table 5.7, we have just lease and retuned vehicles with base contract. In addition, there is no any swap of the vehicles.

Like the previous section the values for the CVaR and expected cost are reduced when ρ is decreased for three values of annual learning rate of technological change for EVs (Figure 5.4). For example, in the case of δ equal to 1.2, when ρ reduces from 0.25 to 0, we have decrease of £2.11M (8%) in CVaR and £1.63M (7%) in expected cost. We have proved this result in Propositions 5.2. and 5.4. In addition, as seen in Figure 5.4, by increasing the learning rate from 1.2 to 2, because we have more EVs in the fleet system and they more expensive than Diesel, the slope of increase in expected cost, when ρ increases from 0 to 0.25, is reduced.

If we also compare the CVaR and expected cost for each corresponding ρ in Figures 5.3 and 5.4, we conclude that we have a reduction in values of CVaR and expected cost for all values ρ, in Figure 5.4. For example, when ρ equals to 0, and δ equal to 1.2, we have £3.03M (11%) reduction in CVaR and £0.83M (4%) in expected cost. Based on your previous we have found that the more EVs in the fleet system the lower CVaR. So, the reduction in CVaR is higher than expected cost. This is also true for other values ρ and δ.

Table 5.5: Optimal policy for lights vans with technological development of EVs, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1), δ=1.2, and ρ=0

			Number of Vehicles				
			Leased			Returned	Swapped
Year	Tech.	age	C1	C2	C3	C2	
2013	D	1		46			
	E	1		5	1026		
2014	D	2				8	
	E	1	110	270	471		
		2					808
2015	D	3				24	
	E	1	47	51	100		
		2					7
		3					171
2016	D	4				9	
	E	1	54	2	1		
		2				1	5
		3				2	3
		4				2	33

Table 5.6: Optimal policy for lights vans with technological development of EVs, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1), δ=1.2, and ρ=0.1

			Number of Vehicles					
			Leased			Returned		Swapped
Year	Tech.	age	C1	C2	C3	C1	C2	
2013	D	1	15	32				
	E	1	105		925			
2014	D	1						
		2				2	21	
	E	1	948					
		2						890
2015	D	3				8	10	
	E	1	59					
		3				7		33
2016	D	4				3	1	
	E	1	21					
		4				19		1

Table 5.7: Optimal policy for lights vans with technological development of EVs, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1), δ=1.2, and ρ=0.25

			Number of Vehicles	
			Leased	Returned
Year	Tech.	age	C1	C1
2013	D	1	45	
	E	1	1032	
2014	D	1		
		2		17
	E	1	51	
2015	D	3		20
	E	1	26	
		3		6
2016	D	4		5
	E	1	19	
		4		19

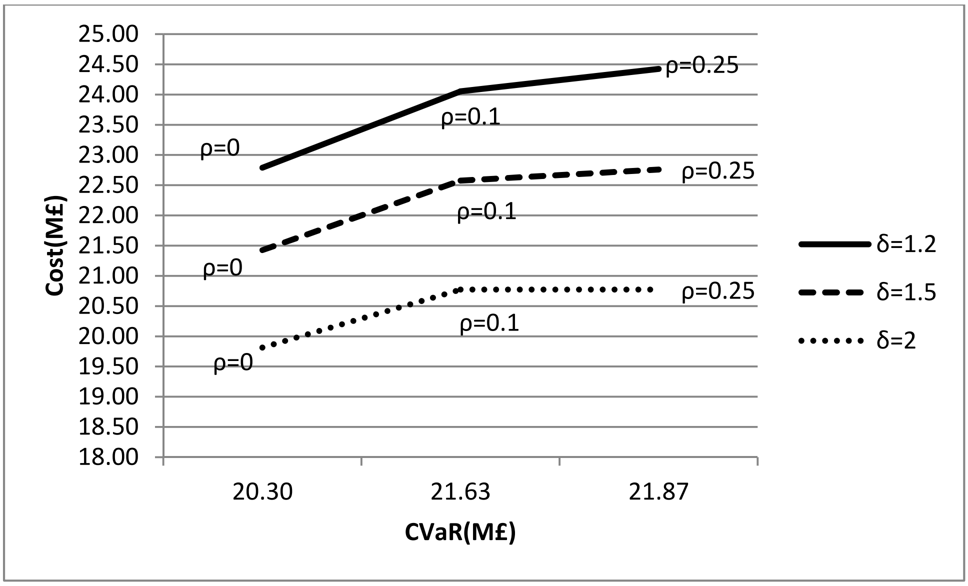

Figure 5.4: The values of CVaR and Expected Cost for Lights Vans with Technological Development of EVs, ω=0.5, 10165 scenarios, θ_c= (1,0.5,0.1), δ=1.2,1.5, and 2

5.7 Summary

The leasing market is an important topic that should be considered for fleet replacement decisions. In terms contribution to the literature of fleet replacement, we have developed a new model in a dynamic setting, for considering the flexibility of different leasing contracts including uncertainties which are CO_2 prices, fuel prices, mileage driven, and fuel consumption using real options methodology CVaR. Our approach is different from Kleindorfer et al. (2012), in terms of stochastic parameters in the model and different options for the leasing contracts and using CVaR.

We have considered three contracts which are base (without option), contract with return option, and contract with option to swap the vehicle. Because the light vans characteristics are between small and medium vans, we selected them for considering the optimal policies. Moreover, for the value of ω, we assumed ω equals to 0.5 in which the weights for expected cost and CVaR in the objective function are equal. In addition, for the value of ρ, the coefficient for relation between value and price of an option, we considered 0%, 10%, and more than 25% of the value the options for base, return, and swap contacts, respectively. Finally, for technological change of EVs we considered the annual learning rate, δ, of 12%. The main results of this chapter are:

1. When the coefficient for relation between value and price of an option (ρ), equals zero, i.e., the options are free, we have different combination of contacts for leasing of vehicles. Moreover, as expected, because the swap contract has the lowest penalty, we have a lot of swapped vehicles. For example, in 2014 we have 847 diesel vehicles at age 2, which are swapped by other contacts with the same technology. However, we do not have any returned vehicles with base contract; because it has the highest penalty.
2. When we change the value of the coefficient for relation between value and price of an option (ρ) to 0.1, i.e., we should pay for return and swap contracts, 10% of their value the results are changed. The vehicles are leased with base contract and contract with swap option. In addition, we have returned vehicles with base contract and contract with swap option. However, there are no returned vehicles with option to return due to the higher penalty of them in comparison with swap contracts. The interesting observation is retuned vehicles with base contract. Indeed, there is a trade-off between penalty and price of an option. Finally, when ρ is 0.25 or more, we have just lease and returned vehicles with base contract. In addition, there is not any swap of the vehicles.
3. For considering the technological change of EVs, the main insights in terms of choosing optimal contracts are similar with the case of without considering the technological development of EVs for different values of the coefficient

for relation between value and price of an option (ρ). However, the optimal technologies are different in two cases. Indeed, EVs are the dominant technology for leasing when the effect of technology is taken into account. However, in the case of considering the model without the technological development of EVs, the dominant chosen technology is Diesels. Petrol technology is selected when we assume there is no technology development of EVs in few cases. Finally, Hybrid technology is not selected as the optimal choice in any cases.

4. The values CVaR and expected cost, when the coefficient for relation between value and price of an option (ρ) equals to zero are less than the case when ρ is not equals to zero. The reason is that using contracts with options decreases the whole CVaR and expected cost. In other words, when all contracts are used in the fleet system, the total cost and CVaR are minimized. Indeed, the lower the price of the options, the lower the CVaR and expected cost. For example, when ρ decreases from 0.25 to 0, we have £3.56M (12%) decrease in CVaR and £2.99M (11%) in expected cost.
5. The values for the CVaR and expected cost are also decreased with all three cases of ρ, when the case of technological change of EVs is considered. For example, in the case of δ equal to 1.2, when ρ reduces from 0.25 to 0, we have decrease of £2.11M (8%) in CVaR and £1.63M (7%) in expected cost. In addition, by increasing the annual learning rate for technological change of EVs from 1.2 to 2, because we have more EVs in the fleet system and they more expensive than Diesel, the slope of increase in expected cost, when ρ increases from 0 to 0.25, is reduced.

6. If we also compare the CVaR and expected cost for each corresponding the coefficient for relation between value and price of an option (ρ) with and without considering the impact of technological change of EVs, we conclude that we have a reduction in values of CVaR and expected cost for all values ρ, when technological change is considered. For example, when ρ equals to 0, and δ equal to 1.2, we have £3.03M (11%) reduction in CVaR and £0.83M (4%) in expected cost. Based on our previous research, we have found that the more EVs in the fleet system the lower CVaR. So, the reduction in CVaR is higher than expected cost. This is also true for other values ρ and annual learning rate (δ) for the technological development of batteries for EVs.

In the next Chapter we draw conclusions from this thesis in the three distinct models analyzed in the previous Chapters. Moroever, we present the contributions of this thesis in terms of policy and methodical implications.

CHAPTER 6

CLOSURE

In this Chapter, the thesis is summarized and the main conclusions are drawn and the main contributions of the thesis are highlighted. First, we explain the main contributions.

6.1. Contributions

The contribution of this thesis has two components, in fleet management policy and in the method used to apply it.

At a policy level, this thesis addresses the effect of adoption of new technological advanced vehicles on the risk and expected cost of the fleet system of the companies. The idea for this issue arises from the need to study the adoption of new technological advanced vehicles in many companies in Europe and in the US, from an economic perspective. Because EVs are still in their infancy in terms of development and because they require a high investment cost, this study addresses this issue from a risk perspective, which has not been considered before.

At a methodological level, this thesis presents three contributions: First, in Chapters 3 and 4, it studies the new formulation of fleet problem by using two stage and multi stage stochastic programming and CVaR, which accounts the uncertainty in the decision process. In other words, one of the contributions of Chapters 3, and 4 is to consider risk and cost minimization by using CVaR, in a stochastic programming model, as part of the objective function of the company, which has not been considered before in the literature. Specifically, because the objective of this stochastic program is to minimize the cost, and risk, simultaneously, we have minimized the weighted average of the total expected cost, and CVaR. That is, by changing an exogenous tradeoff parameter to different combinations of the total expected cost and CVaR, the risks over the planning horizon are minimized, depending on whether the focus is more on cost or on risk.

Second, in Chapter 4, it models a recursive formulation of CVaR, which takes into account the time consistency issue in the dynamic setting. Indeed, Our approach is different from Shapiro (2009, 2011), where cost-to-go functoin concept was used to satisfy the time consistency principle, as we provide a recursive formulation of the CVaR for a scenario tree, explicitly computing the CVaR of the parent node as a function of the CVaR and expected conditional expectations of the extreme cost of the respective children nodes and we name it Recursive Expected CVaR (RECVaR). We have concluded that RECVaR provides more intuitive and robust results, because it takes into account the risks that exist in the middle stages of the scenario tree. However, Shapiro (2011) formulation is not sensitive to these kinds of risks. It also differs from Boda and Filar (2006), in which the target-percentile approach was applied to consider the time consistency rule. So, our methodological contribution in Chapter 4 is suggestion a new formulation of time consistent CVaR.

Third, in Chapter 5, it extends the model in dynamic setting with CVaR for considering flexibility in fleet replacement problem using contracts with different options. Indeed, there is also a gap in the literature which is to view the problem of sustainable fleet replacement, taking into account the inherent flexibility that can exist for leasing contracts due to existence of uncertainties, by using real options analysis. Our approach in this thesis is different from Kleindorfer et al. (2012), in terms of stochastic parameters in the model and different options for the leasing contracts. This is also a new approach in the literature which takes into account the interaction between using contracts and CVaR in the fleet system and has not been analyzed before. In addition, in Chapter 5 the impact of technological change of batteries of EVs is considered for the evaluation of options contracts.

6.2 Concluding Remarks

In all Chapters 3, 4, and 5 the risk drivers which are considered are fuel and CO_2 prices, mileage driven, and fuel consumption. In Chapter 3, we have considered a different distribution for the risk drivers in comparison with Chapters 4 and 5 due to the fact that the models are studied in different settings. Finally, all the vehicles used in the fleet system are leased.

In Chapter 3 we have analyzed the importance of risk drives for EVs and diesel vehicles and comparison of the value of CVaR of EVs with diesel technology for each case of risk driver. The results show that, if each stochastic process is considered separately, the most important risk driver for a diesel vehicle is the mileage driven, followed by fuel consumption, and lastly, fuel prices. For the case of EVs, the first important risk factor is mileage, followed by fuel prices and then CO_2 prices. In addition, for each stochastic process of fuel prices, mileage driven, and fuel consumption, the value of CVaR for EVs is less than for fossil fuel vehicles, under certain conditions.

In Chapter 4 we have considered the risk management issue when different types (capacity) of vehicles with different technologies are used in the fleet system. In this case each vehicle imposes a different risk depending on its characteristic. So, we have categorized (clustered) them based on two important risk drives which are mileage driven and fuel consumption and then we have studied the behavior of each category for each type and technology of vehicles. In addition, we have compared the behavior of each group with the case when they are not clustered (combined cluster). In Chapter 4, we have also considered the Hybrid vehicles. The main findings of model are as follows: a) for clusters with low mileage (500 miles/month) and average mileage (1000 miles/month) with fuel efficiencies of all type of vehicles, diesel is the dominant choice for risk or cost minimisation purposes. b) Petrol is an optimal choice for risk and cost minimisation in the in low and average mileage driven clusters of different type of vehicles. c) The Hybrid-petrol and Hybrid-diesel, which were also considered in the model, cannot compete with diesel technology in terms cost efficiency because of the high leasing/ownership costs. Their penetration ratio in different type of vehicles ranges between 1% and 3%. d) EVs generally contribute to high mileage clusters of different capacities of the vehicles. e) for all capacities, the risk per vehicle is reduced by clustering when compared to combined clusters. However, the expected cost per vehicle is not reduced by clustering for clusters with high mileage profiles.

In Chapter 5, we have studied another important issue which is related to leasing of vehicles. In reality fleet managers can lease vehicles with different options. So, we have extended the model in Chapter 4 by using different option contracts of base (contract with no option), return, and swap options. We have also considered the technological development of batteries of EVs expected during the life of contracts on the optimal replacement policies. The major findings are: a) using contracts with options decreases the whole CVaR and expected cost. In other words, when all contracts are used in the fleet system, the total cost and CVaR are minimized. b) EVs are the dominant technology for leasing when the effect of technology is taken into account. However, in the case of considering the model without the technological development of EVs, the dominant chosen technology is Diesel. Petrol technology is selected when we assume there is no technology development of EVs in few cases.

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